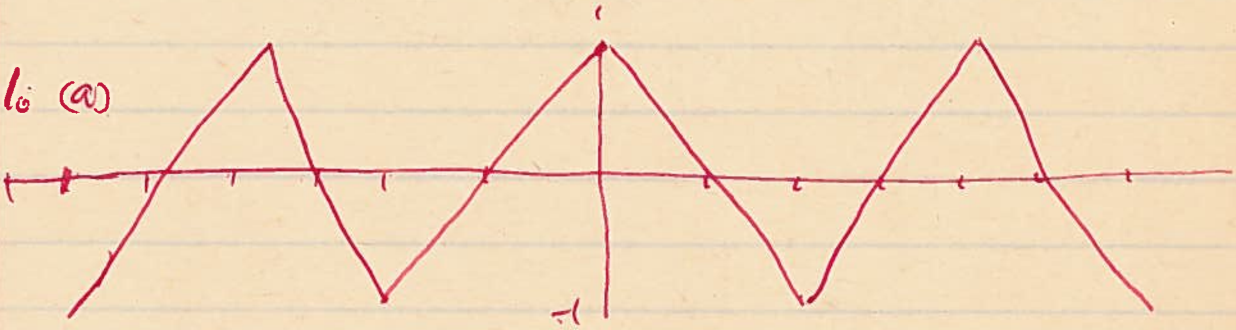
Exam V Key

1. (a)



(b) $f(x)$ is even so only cosine terms appear.

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{2}x$$

$$a_0 = \frac{2}{2}\int_0^2 (1-x)\,dx = -\frac{(1-x)^2}{2}\Big]_0^2 = 0$$

$$a_n = \int_0^2 (1-x)\cos\frac{n\pi x}{2}\,dx$$

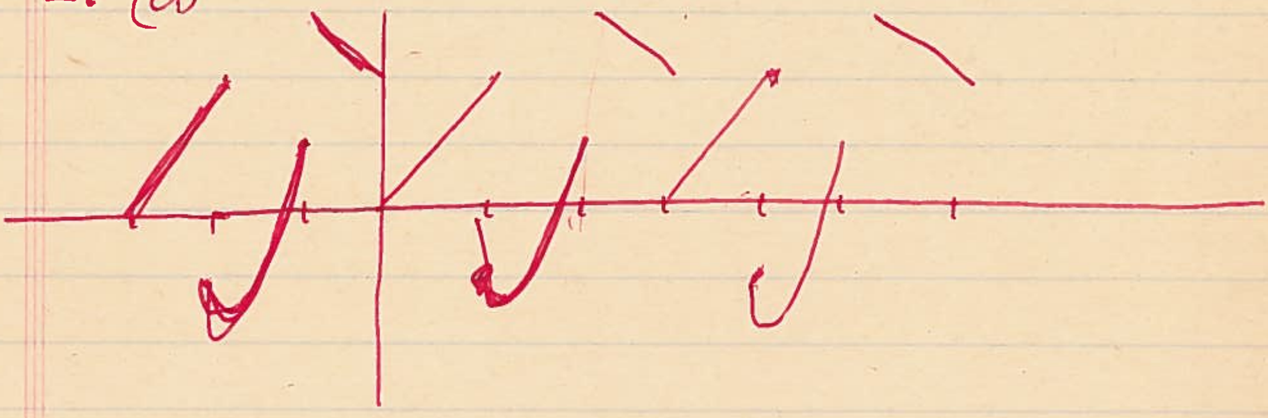
$$= \int_0^2 \cos\frac{n\pi x}{2}\,dx - \int_0^2 x\cos\frac{n\pi}{2}x\,dx$$

$$= \frac{2}{n\pi}\sin\frac{n\pi x}{2}\Big]_0^2 - \left[\frac{4}{n^2\pi^2}\cos\frac{n\pi x}{2} + \frac{2x}{n\pi}\sin\frac{n\pi x}{2}\right]_0^2$$

$$= -\frac{4}{n^2\pi^2}\left[\cos n\pi - 1\right] = \frac{4}{n^2\pi^2}\left(1 + (-1)^{n+1}\right)$$

$$f(x) \sim \frac{4}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{1+(-1)^{n+1}}{n^2}\right]\cos\frac{n\pi}{2}x$$

2. (a)



(b) $g(0) = \frac{1}{2}(2+0) = 1$

$g(2) = \frac{1}{2}(3+1) = 2$

$g(4) = \frac{1}{2}(2+(-1)) = \frac{1}{2}$

$g(-1) = g(2) = 2$

$g(\frac{1}{2}) = 1.$

3. $x^2 = 2\int_0^x g(t)\,dt$

$= 4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^x \sin nt\,dt$

$= 4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left(-\frac{\cos nt}{n}\right)\Big]_0^x$

$= 4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\left(-\frac{\cos nx}{n} + \frac{1}{n}\right)$

$$= 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx + 4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$$

$$\shortparallel$$

$$\frac{a_0}{2}.$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 dx = \frac{1}{\pi}\frac{x^3}{3}\Big]_{-\pi}^{\pi} = 2\frac{\pi^2}{3}.$$

$$\boxed{f(x) \sim \frac{\pi^2}{3} + 4\sum_{n=0}^{\infty}\frac{(-1)^n}{n^2}\cos nx}$$

(b) $f_1(x) = f'(x)$

$$= \frac{1}{2}\cos x + \frac{2}{\pi}\sum_{n=\text{even}}\frac{n\sin nx}{n^2-1}.$$

4. $y_p = \sum_{n=1}^{\infty}(A_n \cos n\pi x + B_n \sin n\pi x.)$

$y_p'' + 10y_p$

$$= \sum_{n=1}^{\infty}(-n^2\pi^2 A_n + 10A_n)\cos n\pi x + (-n^2\pi^2 B_n + 10B_n)\sin n\pi x$$

$$= \sum \frac{\cos n\pi x}{n^2} + \frac{\sin n\pi x}{n}$$

$$\Longrightarrow (10-n^2\pi^2)A_n = \frac{1}{n^2} \Rightarrow A_n = \frac{1}{n^2(10-n^2\pi^2)}$$

$$(10-n^2\pi^2)B_n = \frac{1}{n} \qquad B_n = \frac{1}{n(10-n^2\pi^2)}.$$

$\Rightarrow$

$$Y_p = \sum_{n=1}^{\infty} \left(\frac{\cos n\pi x}{n^2(10-n^2\pi^2)} + \frac{\sin n\pi x}{n(10-n^2\pi^2)} \right)$$