Math 2070 Sec. 1 Exam V December 6, 1994

Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. The value of each problem is shown in parentheses to the left of the problem. Some indefinite integrals which may be useful are given at the end of the exam.

Please *print* your name and student number in the space provided below, and turn in this sheet with your solution papers.

Name:

Student Number:

(30) 1. Let $f(x) = \begin{cases} 0 & \text{if } -\pi < x < 0 \\ x & \text{if } 0 \le x < \pi \end{cases}$. Sketch the graph of f and compute the Fourier series of f.

(30) 2. Let $f(x) = \begin{cases} x^2 & \text{if } 0 < x < 2 \\ x - 4 & \text{if } 2 < x < 4 \end{cases}$.

(a) Let f_1 be the periodic extension of $f(x)$ to all of $\mathbf{R}$, assuming a period of 4. Sketch the graph of f_1 on $[-4, 4]$. What is the natural way to define f_1 at 2 so that the Fourier series of f_1 converges to f_1?

(b) Sketch the graph of the odd extension f_2 of f to the interval $[-4, 4]$.

(c) Sketch the graph of the even extension f_3 of f to the interval $[-4, 4]$.

(20) 3. The Fourier series expansion of $f(x) = x^2$ (on $-\pi \le x \le x$) is

$$(*) \qquad f(x) = x^2 = \frac{\pi^2}{3} - 4\left(\cos x - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \frac{\cos 5x}{5^2} + \cdots\right)$$
$$= \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{\cos nx}{n^2}.$$

(a) Using formula (*), compute the Fourier series of $g(x) = x$.

(b) Using formula (*) and part (a), compute the Fourier series of $h(x) = x^3$.

(20) 4. Consider the differential equation

$$(1) \qquad y'' + 2y' + 3y = f(t)$$

where $f(t)$ is periodic of period 2π with Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

Let y_p be the periodic solution of Equation (1), with Fourier series

$$y_p = \frac{A_0}{2} + \sum_{n=1}^{\infty}(A_n \cos nx + B_n \sin nx) = \sum_{n=0}^{\infty} y_n.$$

Assuming that $a_2 = 0$ and $b_2 = 5$, write the differential equation y_2 and then solve it to compute A_2 and B_2.

The following integrals may be of use:

$$\int x \sin ax\, dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + C$$
$$\int x \cos ax\, dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C$$