Math 2070 Sec. 1 Final Exam December 17, 1994

Instructions. Answer each of the questions on your own paper. Be sure to show your work so that partial credit can be adequately assessed. Read each problem carefully, and do exactly what is requested. A table of basic Laplace transforms is appended to the exam. The value of each problem is shown in parentheses to the left of the problem.

Please *print* your name and student number in the space provided below, and turn in this sheet with your solution papers.

Name:

Student Number:

In exercises 1 to 10, solve the given differential equation or initial value problem by whatever method is appropriate.

(12) 1. $y' = 8e^{2x} - 3y$

(12) 2. $3y' = \dfrac{4x}{y^2}$

(12) 3. $y'' - 14y' + 49y = 0$

(12) 4. $y'' - y' - 2y = 5e^{4x} + 6x$

(12) 5. $y'' + 6y' + 13y = 0$

(12) 6. $y^{(4)} - 8y'' + 16y = 0$

(12) 7. $y'' + 16y = \delta(t-3) \qquad y(0) = 1,\ y'(0) = -1$

(16) 8. $y'' + 5y = \displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\cos\frac{n\pi}{2}x - \frac{1}{n^4}\sin\frac{n\pi}{2}x\right)$

(16) 9. $\mathbf{x}' = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\mathbf{x}$

(16) 10. $\mathbf{x}' = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}\mathbf{x}$

(18) 11. (a) Compute the inverse Laplace transform of the function $\dfrac{s-3}{s^2+10s+9}e^{-2s}$.

(b) Let $f(t) = \begin{cases} 0 & \text{if } 0 \le t < 2 \\ t-1 & \text{if } 2 \le t < 3 \\ -4 & \text{if } t \ge 3 \end{cases}$. Sketch the graph of $f(t)$ and compute the Laplace transform of $f(t)$.

(18) 12. Let $f(x)$ be a periodic function of period π. The formula for $f(x)$ on the interval $[0, \pi)$ is
$$f(x) = \begin{cases} 0 & \text{if } 0 \le x < \pi/4 \\ 5 & \text{if } \pi/4 \le x < 3\pi/4 \\ 0 & \text{if } 3\pi/4 \le x < \pi \end{cases}.$$

(a) Sketch the graph of $f(x)$ on the interval $[-2\pi, 2\pi]$.

(b) Compute the Fourier series of $f(x)$.

(c) Let $g(x)$ be the sum of the Fourier series found in part (b). Compute $g(0)$, $g(\pi/4)$, and $g(10)$.

(12) 13. Find all eigenvalues of the matrix

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 2 & 2 \\ -1 & -1 & 0 \end{bmatrix}.$$

(12) 14. The eigenvalues of the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix}$$

are 1, -1, and -2. Find all the eigenvectors with the eigenvalue -2.

(12) 15. A force of 2 lb stretches a spring 1 ft. A 3.2-lb weight is attached to the spring and the system is then immersed in a medium that imparts a damping force equal to 0.4 times the instantaneous velocity.

(a) Write the differential equation governing such a system assuming that there is no externally applied force.

(b) Find the equation of motion if the weight is released from rest 1 ft above the equilibrium position.

(c) Express the equation of motion in phase-amplitude form.

Basic Laplace Transforms

Function	Transform
e^{at}	$\dfrac{1}{s-a}$
$u_c(t)$	$\dfrac{e^{-cs}}{s}$
t^n	$\dfrac{n!}{s^{n+1}}$
$\sin bt$	$\dfrac{b}{s^2+b^2}$
$\cos bt$	$\dfrac{s}{s^2+b^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t)$	$(-1)^n \dfrac{d^n F}{ds^n}(s)$
$f^{(n)}(s)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$\dfrac{f(t)}{t}$	$\int_s^\infty f(r)\,dr$
$\int_0^t f(r)\,dr$	$\dfrac{F(s)}{s}$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$\delta(t-c)$	e^{-cs}