Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. [**10 Points**] Let $A = \begin{bmatrix} 8 & x \\ 3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$. Determine all values of x for which $AB = BA$.

2. [**5 Points**] Let A be a 3×1 matrix, B a 2×3 matrix, and C a 2×1 matrix. which of the following matrix products is well defined and the result is a 1×2 matrix?
   (a) AB
   (b) $B^T C$
   (c) $C^T BA$
   (d) BAC^T
   (e) $(BA)^T$

3. [**20 Points**] Find all solutions of the following system of linear equations. Be sure to show all your steps!
$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & + & x_4 & = & 8 \\ x_1 & + & 3x_2 & & & + & x_4 & = & 7 \\ x_1 & & & + & 2x_3 & + & x_4 & = & 3 \end{array}$$

4. [**20 Points**] Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$.
   (a) Compute A^{-1}.
   (b) Using your answer to part (a), solve the system of equations
$$\begin{array}{rcrcrcl} x_1 & - & x_2 & & & = & 1 \\ 2x_1 & & & + & x_3 & = & 1 \\ & & x_2 & - & x_3 & = & 1 \end{array}$$

5. [**15 Points**] Find conditions that b_1, b_2, and b_3 must satisfy for the following system to be consistent:
$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & - & 3x_3 & = & b_1 \\ 2x_1 & + & 3x_2 & + & 3x_3 & = & b_2 \\ 5x_1 & + & 9x_2 & - & 6x_3 & = & b_3 \end{array}$$

6. [**10 Points**] Find all the values of y for which the matrix

$$A = \begin{bmatrix} 2 & 1 & 3y & 4 \\ 0 & y-1 & 4 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & y & 4 \end{bmatrix}$$

is **not** invertible.

7. [**10 Points**] Let A be an $n \times n$ invertible matrix. Determine whether each of the following statements is True (**T**) or False (**F**)?

   (a) $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

   (b) A is a product of elementary matrices.

   (c) $\det A = 0$.

   (d) The reduced row echelon form of A is the identity matrix I_n.

   (e) $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $n \times 1$ matrix $\mathbf{b}$.

8. [**10 Points**] Suppose that A is the 3×3 matrix

$$\begin{bmatrix} 1 & 8 & 3 \\ x & y & z \\ -3 & 7 & 2 \end{bmatrix}.$$

Assuming that $\det(A) = 5$, compute the determinant of the following matrices:

$$\text{(a) } B = 2A^2; \qquad \text{(b) } C = \begin{bmatrix} x & y & z \\ 1 & 8 & 3 \\ 6+4x & -14+4y & -4+4z \end{bmatrix}.$$