**Instructions.** Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. [**12 Points**] Which of the following subsets of $\mathbb{R}^3$ are subspaces. Give a reason in each case.

   (a) $S_1 = \{(a,\, b,\, c) : a + b + c = 0\}$.

   (b) $S_2 = \{(a,\, b,\, c) : abc = 0\}$.

2. [**16 Points**] Find a basis for and the dimension of the solution space of the homogeneous system

$$\begin{array}{rcrcrcrcl} x_1 & - & 3x_2 & & & - & 5x_4 & = & 0 \\ -x_1 & + & 4x_2 & + & x_3 & + & 7x_4 & = & 0 \\ 2x_1 & + & x_2 & + & 7x_3 & + & 4x_4 & = & 0 \\ 2x_1 & - & 2x_2 & + & 4x_3 & - & 2x_4 & = & 0 \end{array}$$

   You may use the fact that the coefficient matrix of this system is the matrix $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ -1 & 4 & 1 & 7 \\ 2 & 1 & 7 & 4 \\ 2 & -2 & 4 & -2 \end{bmatrix}$ and the matrix $R = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is the reduced row-echelon matrix associated to $A$.

3. [**16 Points**] Let $\mathbf{v}_1 = (-1,\, 2,\, 3,\, 1)$, $\mathbf{v}_2 = (5,\, 0,\, 2,\, -3)$, and $\mathbf{v}_3 = (3,\, 4,\, 8,\, 1)$. Determine if the set $S = \{\mathbf{v}_1,\, \mathbf{v}_2,\, \mathbf{v}_3\} \subseteq \mathbb{R}^4$ is linearly independent. If it is not linearly independent, find an explicit liner combination $k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + k_3\mathbf{v}_3 = \mathbf{0}$ with at least one scalar $k_i \neq 0$.

4. [**16 Points**] Let $W = \mathrm{Span}\,\{\mathbf{v}_1,\, \mathbf{v}_2,\, \mathbf{v}_3\}$ where $\mathbf{v}_1 = (1,\, -1,\, 2)$, $\mathbf{v}_2 = (3,\, -2,\, 5)$, and $\mathbf{v}_3 = (2,\, -1,\, 3)$.

   (a) Find a subset $S \subset \{\mathbf{v}_1,\, \mathbf{v}_2,\, \mathbf{v}_3\}$ that forms a basis of $W$. What is the dimension of $W$?

   (b) Write each vector $\mathbf{v}_i$ that is not in $S$ as a linear combination of the vectors in $S$.

5. [**12 Points**] Let $S = \{\mathbf{v}_1,\, \mathbf{v}_2,\, \mathbf{v}_3\}$ be the basis of $\mathbb{R}^3$ with $\mathbf{v}_1 = (1,\, 0,\, 0)$, $\mathbf{v}_2 = (2,\, 2,\, 0)$, and $\mathbf{v}_3 = (3,\, 3,\, 3)$. If $\mathbf{v} = (2,\, -1,\, 3)$ compute the coordinate vector $(\mathbf{v})_S$ of $\mathbf{v}$ relative to the basis $S$.

6. [**12 Points**] Let $\mathbf{u} = (-3,\, 2,\, 1,\, 0)$ and $\mathbf{a} = (0,\, 2,\, -1,\, 1)$ be vectors in $\mathbb{R}^4$. Compute the following:

   (a) The vector component $\mathrm{proj}_{\mathbf{a}}\,\mathbf{u}$ of $\mathbf{u}$ along $\mathbf{a}$.

   (b) The vector component of $\mathbf{u}$ orthogonal to $\mathbf{a}$.

   (c) The cosine of the angle between $\mathbf{u}$ and $\mathbf{a}$.

7. [**16 Points**] Complete the following definitions:

   (a) A vector $\mathbf{w}$ is a *linear combination* of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ if ...

   (b) A set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of vectors in a vector space $V$ is *linearly dependent* if ...

   (c) A *basis* of a vector space $V$ is a set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of vectors such that ...

   (d) The *dimension* of a vector space $V$ is ...