Exam 2 will be on Wednesday, October 20, 2010. The syllabus for Exam 2 consists of Sections 3.1, 3.2, 3.3 and 4.1 to 4.5. You should know the main definitions, results and computational techniques that we have covered in these sections. Here is a short list of particularly important problems that have been previously assigned: Section 3.2: 19, 23; Section 3.3: 5, 23, 27; Section 4.2: 3, 7, 9; Section 4.3: 7; Section 4.4: 3, 9, 11; Section 4.5: 1, 3, 5, 13. Here are some additional exercises of a similar nature that have appeared on previous exams.

You may also review Exam 2 from Summer 2008, but note that questions 2 and 5 of that exam cover topics in chapter 4 that we have not yet covered.

1. Which of the following subsets of $\mathbb{R}^3$ are subspaces? Give a reason in each case.
   (a) $S_1 = \{(a,\, b,\, c) : a = 0\}$.
   (b) $S_2 = \{(a,\, b,\, c) : ab = 0\}$.
   (c) $S_3 = \{(a,\, b,\, c) : a + b = 0\}$.
   (d) $S_4 = \{(a,\, b,\, c) : a = 1\}$.

2. Let $A = \begin{bmatrix} 1 & -3 & 0 & -5 \\ -1 & 4 & 1 & 7 \\ 2 & 1 & 7 & 4 \\ 2 & -2 & 4 & -2 \end{bmatrix}$ and let $R = \begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Then $R$ is the reduced row-echelon matrix associated to $A$. Compute a basis for the solution space of the homogeneous equation $Ax = 0$.

3. Let $\underline{u} = (-1,\, 2,\, 3,\, 1)$, $\underline{v} = (5,\, 0,\, 2,\, -1)$, and $\underline{w} = (3,\, 4,\, 8,\, 1)$. Determine if the set $S = \{\underline{u},\, \underline{v},\, \underline{w}\} \subset \mathbb{R}^4$ is linearly independent. If it is not linearly independent, find an explicit linear combination $k_1\underline{u} + k_2\underline{v} + k_3\underline{w} = 0$ with at least one scalar $k_i \neq 0$.

4. Let $W = \mathrm{Span}\{\underline{v}_1,\, \underline{v}_2,\, \underline{v}_3\}$ where $\underline{v}_1 = (1,\, -1,\, 2)$, $\underline{v}_2 = (3,\, -2,\, 5)$ and $\underline{v}_3 = (2,\, -1,\, 3)$.
   (a) Find a subset $S \subseteq \{\underline{v}_1,\, \underline{v}_2,\, \underline{v}_3\}$ which forms a basis of $W$.
   (b) Write each $\underline{v}_i$ which is not in $S$ as a linear combination of the vectors in $S$.

5. Let $S = \{\underline{v}_1,\, \underline{v}_2\}$ be the basis of $\mathbb{R}^2$ with $\underline{v}_1 = (1,\, 1)$ and $\underline{v}_2 = (1,\, -1)$.
   (a) If $\underline{v} = (4,\, 2)$ compute the coordinate vector $(\underline{v})_S$.
   (b) If $(\underline{w})_S = (1,\, -4)$, then find $\underline{w}$.

6. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22}$ is a $2 \times 2$ matrix then we define the trace of $A$, denoted $\mathrm{Tr}(A)$ to be the real number $a + d$, i.e., the trace of a matrix $A$ is the sum of the diagonal entries of $A$. Let $V = \{A \in M_{22} : \mathrm{Tr}(A) = 0\}$. Then $V$ is a subspace of $M_{22}$. Find a basis of $V$. What is the dimension of $V$?