Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. [**20 Points**] Let $A = \begin{bmatrix} 1 & 2 & -2 & 0 & 7 \\ -2 & -3 & 1 & -1 & -5 \\ -3 & -4 & 0 & -2 & -3 \\ 3 & 6 & -6 & 5 & 1 \end{bmatrix}$ and let $R = \begin{bmatrix} 1 & 0 & 4 & 0 & -3 \\ 0 & 1 & -3 & 0 & 5 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Then R is the reduced row-echelon matrix associated to A. You may assume this as given. *It is not necessary to check it.* Using this fact, answer the following questions.

   (a) Compute a basis for the row space Row(A) of A.

   (b) Compute a basis for the column space Col(A) of A.

   (c) Compute a basis for the nullspace Null(A) of A.

   (d) What is the rank of A?

2. [**7 Points**] Is the vector $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ an eigenvector of the matrix $A = \begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$? If so, what is the corresponding eigenvalue?

3. [**12 Points**] Determine the characteristic polynomial and the eigenvalues of the matrix

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 0 & 5 & -6 \\ 0 & -1 & 4 \end{bmatrix}.$$

   You are *not* asked to find any eigenvectors.

4. [**16 Points**] Find an invertible matrix P and diagonal matrix D such that $P^{-1}AP = D$, where the matrix $A = \begin{bmatrix} 2 & 10 \\ 1 & -1 \end{bmatrix}$ and you may assume that the eigenvalues of A are $\lambda = 4$ and $\lambda = -3$.

5. [**10 Points**] Let $A = \begin{bmatrix} 5 & -12 \\ 1 & -2 \end{bmatrix}$ and $P = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$. Then $P^{-1}AP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$. (You do not need to check this.) Using this information, compute A^8.

6. [**12 Points**] Determine if the following functions are linear transformations. Give reasons.

   (a) $T : M_{22} \to \mathbb{R}$ defined by $T(A) = \det A$.

   (b) $S : P_1 \to \mathbb{R}$ defined by $S(a + bx) = a + 2b$.

7. [**14 Points**] Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ be the basis of $\mathbb{R}^2$ with $\mathbf{v}_1 = (2, 1)$ and $\mathbf{v}_2 = (1, 1)$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation for which $T(\mathbf{v}_1) = (1, 0)$ and $T(\mathbf{v}_2) = (-1, 1)$. Find a formula for $T(x_1, x_2)$ and use it to compute $T(3, 5)$.

8. **[9 Points]** Let A be a 7×6 matrix. and assume that the dimension of the nullspace of A is 2.

   (a) What is the dimension of the column space of A?

   (b) What is the rank of A^T? (Recall that A^T is the transpose of A.)

   (c) What is the nullity of A^T