Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper. There are 5 problems, with a total of 100 points possible. The points for each problem is listed in parentheses.

(30) 1. Consider the system of linear equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= a \\ 4x_1 + 5x_2 + 6x_3 &= b \\ 7x_1 + 8x_2 + 9x_3 &= c \end{aligned} \tag{1}$$

where a, b, and c are some real numbers.

(a) Find the general solution of the system when $a = 0$, $b = 0$, $c = 0$.

(b) A particular solution of Equation (1) when $a = 4$, $b = 10$, $c = 16$ is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

Find the general solution of Equation (1) when $a = 4$, $b = 10$, $c = 16$.

(18) 2. In each part suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row echelon form. Solve the system

(a) $\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & 4 \\ 0 & 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ (b) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 8 \end{array}\right]$

(20) 3. Determine, with justification, whether each subset of $\mathbb{R}^3$ is a vector subspace.

(a) $V_1 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \, a = b + c \text{ and } a,\, b,\, c \in \mathbb{R} \right\}$

(b) $V_2 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \, a = b + 1 \text{ and } a,\, b,\, c \in \mathbb{R} \right\}$

(16) 4. Consider the following vectors $\vec{v_1}$, $\vec{v_2}$, $\vec{v_3}$, and $\vec{w}$ in $\mathbb{R}^3$:

$$\vec{v_1} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} 3 \\ -1 \\ -3 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}.$$

(a) Determine if the vector $\vec{w}$ is in the span of the set $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$.

(b) If $\vec{w}$ is in the span of the set $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$, express $\vec{w}$ as a linear combination of $\vec{v_1}$, $\vec{v_2}$, and $\vec{v_3}$. You should be able to answer this using the work done in part (a).

(16) 5. Determine if the following sets of vector are linearly independent. If **not**, write one vector as a linear combination of other vectors in the set.

(a) $\left\{ \vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} 0 \\ 2 \\ -10 \end{bmatrix} \right\}$

(b) $\left\{ \vec{w_1} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} -6 \\ 2 \\ -6 \end{bmatrix} \right\}$