Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper. There are 5 problems, with a total of 70 points possible. The points for each problem is listed in parentheses. There is an additional 10 point bonus problem.

(10) 1. Use these matrices for this exercise.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 2 \\ -1 & 0 \\ 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$$

Compute each of the following matrices, or explain why the combination is not valid.

(a) $AB + BA$ (b) $2AB + C^2$.

(30) 2. Let $h : \mathcal{P}_2 \to \mathcal{M}_{2\times 2}$ be the homomorphism represented with respect to the basis $\mathcal{B} = \langle 1,\, x,\, x^2 \rangle$ of $\mathcal{P}_2$ and the basis

$$\mathcal{D} = \left(\vec{v}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 & 0) \\ 1 & 0 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

of $\mathcal{M}_{2\times 2}$ by the matrix

$$H = \mathrm{Rep}_{\mathcal{B},\mathcal{D}}(h) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & -2 \\ 2 & 0 & 4 \\ 0 & 1 & -1 \end{bmatrix}.$$

(a) Find $h(\vec{v})$ for $\vec{v} = 2 + x - x^2$.

(b) Find $h(\vec{w})$ for the general vector $\vec{w} = a + bx + cx^2$ in $\mathcal{P}_2$.

(c) Find a basis for the range of h. What is the rank of h?

(d) Is $A = \begin{bmatrix} 3 & -4 \\ 6 & 2 \end{bmatrix}$ in the range of h? What about $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$?

(e) Find a basis for the null space of h. What is the nullity of h?

(f) Is $p_1(x) = 1 + x - 2x^2$ in the null space of h? What about $p_2(x) = 2 - x - x^2$?

(10) 3. Let $A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 4 & 5 \\ 5 & 4 & 0 & 3 \\ 2 & 0 & 0 & 1 \end{bmatrix}$. Compute the determinant of A. Be sure to show all of your work.

(10) 4. Find the inverse, if possible, of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -2 \\ 0 & 1 & 1 \end{bmatrix}$.

(10) 5. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. If $\det A = 5$, compute the determinant of each of the following matrices.

(a) $B = -2A^2$ (b) $C = \begin{bmatrix} a & b & c \\ 2d - 3g & 2e - 3h & 2f - 3i \\ g & h & i \end{bmatrix}$.

Bonus. Let A and B be 2×2 matrices with $AB = 0$. Determine if each of the following statements is **True** or **False**. If True, explain why. If False, provide a counterexample.

(a) $A = 0$ or $B = 0$.

(b) If A is invertible, then $B = 0$.

(c) $BA = 0$.