Exam 3 will be on Wednesday November 15, 2017. The syllabus for Exam 3 consists of Sections Three.III, Three.IV, Three.V.1, Four.I and Four.iii. You should know the main definitions, results and computational techniques that we have covered in these sections. Of The following are problems similar to those you might expect on your exam. Problems can also be similar to assigned homework problems and suggested problems from the text, so you should certainly review those. You may, and are encouraged to, bring any questions that you have to be discussed during class on Monday, November 13.

1. Let $T$ represent a map $t\colon \mathbb{R}^2 \to \mathbb{R}^2$ with respect to the standard bases.

$$T = \begin{bmatrix} 2 & -1 \\ 6 & -3 \end{bmatrix}$$

   Find the range space and null space of $t$.

2. Find the determinant.

$$\begin{vmatrix} 1 & 0 & 2 & 0 \\ 2 & 1 & 1 & 1 \\ 0 & -1 & 1 & -1 \\ 2 & 2 & 1 & 0 \end{vmatrix}$$

3. Represent the linear map $d/dx\colon \mathcal{P}_4 \to \mathcal{P}_3$ with respect to $\mathcal{B} = \langle 1, x, x^2, x^3, x^4 \rangle$ and $\mathcal{D} = \langle 1, 1+x, 1+x+x^2, 1+x+x^2+x^3 \rangle$.

4. Let $T : \mathcal{M}_{2\times 2} \to \mathcal{M}_{2\times 2}$ be defined by $T(A) = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} A$. Using the basis

$$\mathcal{B} = \left( \vec{v}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 & 0) \\ 1 & 0 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

   for $\mathcal{M}_{2\times 2}$, find the following.

   (a) The matrix $\text{Rep}_{\mathcal{B},\mathcal{B}}(T)$ representing $T$ with respect to the bases $\mathcal{B}$ on the domain and $\mathcal{B}$ on the codomain.

   (b) A basis for the null space of $T$.

   (c) A basis for the range of $T$.

5. Let $L : \mathcal{M}_{2\times 2} \to \mathcal{M}_{2\times 2}$ be defined by $L(A) = A^T$, the *transpose* of the matrix $A$. Using the basis

$$\mathcal{B} = \left( \vec{v}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 & 0) \\ 1 & 0 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

   for $\mathcal{M}_{2\times 2}$, find the matrix representation of $L$ using the basis $\mathcal{B}$ on both the domain and codomain.

6. Let $T : P_2 \to P_2$ be defined by $T(f(x)) = 2f''(x) - f(x)$. Use $\mathcal{B} = \langle 1,\, x,\, x^2 \rangle$ as a basis for $P_2$.

   (a) Find the matrix $\text{Rep}_{\mathcal{B},\mathcal{B}}(T)$ of $T$ with respect to the basis $\mathcal{B}$ on both the domain and codomain.

   (b) Find $\det(\text{Rep}_{\mathcal{B},\mathcal{B}}(T))$.

   (c) Is $T$ an isomorphism? Why or why not?

7. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$. Evaluate $\det A$ using row-reduction, showing all of your work.

8. Let $A$ be the $3 \times 3$ matrix expressed in rows as $A = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{R}_2 \\ \mathbf{R}_3 \end{bmatrix}$, and let $B$ be described in terms of the rows of $A$ as $B = \begin{bmatrix} \mathbf{R}_1 + \mathbf{R}_2 + \mathbf{R}_3 \\ 2\mathbf{R}_3 \\ 3\mathbf{R}_2 + \mathbf{R}_3 \end{bmatrix}$. If $\det A = 5$, find $\det B$.

9. Let $\mathcal{B} = \left( \vec{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right)$ be a basis of $R^2$. Let $A = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$ and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by multiplication by $A$: $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^2$. Compute the matrix $\text{Rep}_{\mathcal{B},\mathcal{B}}(T)$ representing $T$ with respect to the bases $\mathcal{B}$ on the domain and $\mathcal{B}$ on the codomain.

10. Determine if each of the following statements is True or False. Explain why.

    (a) Let $V$ be the space of all functions from $\mathbb{R}$ to $\mathbb{R}$ that have infinitely many derivatives. The function $T : V \to V$ given by $T(v) = 3f' - 2f''$ is a linear transformation.

    (b) If the determinant of a $4 \times 4$ matrix is 4, then the rank of the matrix must be 4.

    (c) $\det(A + B) = \det A + \det B$.

    (d) $\det(kA) = k \det A$.

    (e) $\det AB = \det A \det B$.

    (f) If all entries of the diagonal of a matrix $A$ are 0, then $\det A = 0$.

    (g) $(AB)^{-1} = A^{-1}B^{-1}$

11. If $A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$, find $A^{-1}$.

12. Let $A = \begin{bmatrix} 2 & 3 & 0 & 2 \\ 4 & 3 & 2 & 1 \\ 8 & 3 & 0 & 5 \\ 7 & 0 & 0 & 4 \end{bmatrix}$. Use Laplace's cofactor expansion to compute $\det A$.

13. Find all values of $\lambda$ for which the matrix $\begin{bmatrix} \lambda & 1 & -1 \\ 1 & 2 & -2 \\ -1 & 1 & 0 \end{bmatrix}$ is not invertible.