In all homework problems, it is not sufficient to show only the answers. *You must show your work.*

1. Solve the linear system via Gauss-Jordan reduction. Express your answers in the form of Theorem 3.1, Page 24 of the text.

   (a) $$\begin{aligned} 2x + y + 3z &= 1 \\ 4x + 3y + 5z &= 1 \\ 6x + 5y + 5z &= -3 \end{aligned}$$

   (b) $$\begin{aligned} x_1 + 3x_2 + x_3 + 5x_4 + x_5 &= 5 \\ x_2 + x_3 + 2x_4 + x_5 &= 4 \\ 2x_1 + 5x_2 + 7x_4 + x_5 &= 3 \end{aligned}$$

2. Consider the system of linear equations

   $$\begin{aligned} x + y - z &= a \\ x - y + 2z &= b \end{aligned} \tag{1}$$

   where a and b are some real numbers.

   (a) Find the general solution of the associated homogeneous equation.

   (b) A particular solution of Equation (1) when $a = 1$ and $b = 2$ is $(x, y, z) = (1, 1, 1)$. Find the general solution of Equation (1).

   (c) Find a particular solution of Equation (1) when $a = -1$ and $b = -2$.

   (d) Find a particular solution of Equation (1) when $a = 3$ and $b = 6$.

   [Remark: After you have done part (a), it is possible to immediately write down the solutions in the remaining parts. Of course, you need to explain how you got your answers.]

3. Consider the system of linear equations

   $$\begin{aligned} x + y + kz &= 1 \\ x + ky + z &= 1 \\ kx + y + z &= 1 \end{aligned} \tag{2}$$

   where k is some real number. For what value(s) of k does this system have

   (a) a unique solution?

   (b) no solution?

   (c) infinitely many solutions?

   (Justify your assertions.)