In all homework problems, it is not sufficient to show only the answers. *You must show your work.*

1. Determine whether each subset of $\mathcal{M}_{2\times 2}$ is a vector subspace by checking whether condition (2) of Lemma 9 Page 92 is satisfied for each subset.

   (a) V_1 is the collection of 2×2 matrices with 0 in the upper right entry. That is

   $$V_1 = \{ \begin{pmatrix} a & 0 \\ c & d \end{pmatrix} \mid a,\, c,\, d \in \mathbb{R} \}$$

   (b)

   $$V_2 = \{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a,\, b,\, c,\, d \in \mathbb{R} \text{ and } a + d = 1 \}$$

2. Determine (with justification) if each set is linearly independent (in the natural vector space).

   (a) $\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \}$

   (b) $\{ (1\ \ 3\ \ 1), (-1\ \ 4\ \ 3), (-1\ \ 11\ \ 7) \}$

   (c) $\{ \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ -1 & 4 \end{pmatrix} \}$

3. Determine if the given vector, is in the span of the given set, inside the given vector space.

   (a) $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \quad \{ \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \}, \quad \text{in } \mathbb{R}^3$

   (b) $x^2 - 4x^3$, $\{ x^2,\, 2x + x^2,\, x + x^3 \}$, in $\mathcal{P}_3$.

4. Determine which of these sets spans $\mathbb{R}^3$. That is, which of the sets has the property that any vector in $\mathbb{R}^3$ can be expressed as a linear combination of the elements of the set?

   (a) $\{ \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \}$

   (b) $\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \}$