In all homework problems, it is not sufficient to show only the answers. *You must show your work.* These exercises are based on Chapter Two.III.1-Two.III.2 from the text.

1. Find a basis for each vector space. Verify that it is a basis.
   (a) The subspace $M = \{a + bx + cx^2 + dx^3 \mid a - 2b + c - d = 0\}$ of $\mathcal{P}_3$.
   (b) The subspace W of $\mathcal{M}_{2\times2}$:
   $$W = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a - c = 0\}$$
   (c) The subspace V of $\mathbb{R}^4$:
   $$V = \{\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 \mid x - w + z = 0\}$$
   (d) The subspace $N = \{a_0 + a_1x + a_2x^2 + a_3x^3 \mid a_0 + a_1 = 0 \text{ and } a_2 - 2a_3 = 0\}$ of $\mathcal{P}_3$
2. Give two different bases for $\mathbb{R}^3$. Verify that each is a basis.
3. Represent the vector with respect to each of the two bases.
   $$\vec{v} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} \quad B_1 = \langle \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle,\ B_2 = \langle \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \rangle$$