In all homework problems, it is not sufficient to show only the answers. *You must show your work.* These exercises are based on Chapter Two.III.3 from the text.

1. Let $A = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 1 \\ -1 & 2 & 7 \end{bmatrix}$.

   (a) Find a basis for the row space of the matrix A.

   (b) Find a basis for the column space of the matrix A.

   (c) Find a basis for the null space of the matrix A. (Recall that the null space of A is the solution space of the homogeneous linear system $A\vec{x} - \vec{0}$.)

   (d) Determine if each of the vectors $\vec{v} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix}$ is in the row space of A.

   (e) Determine if each of the vectors $\vec{a} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ is in the column space of A.

2. In each part (a)–(b) assume that the matrix A is row equivalent to the matrix B. Without additional calculations, list rank(A) and dim(Nullspace(A)). Then find bases for Colspace(A), Rowspace(A), and Nullspace(A).

   (a) $A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 2 & 6 & -6 & 6 & 3 & 6 \\ -2 & -3 & 6 & -3 & 0 & -6 \\ 4 & 9 & -12 & 9 & 3 & 12 \\ -2 & 3 & 6 & 3 & 3 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

3. Answer each of the following questions related to the rank of an $m \times n$ matrix A.

   (a) If a 4×7 matrix A has rank 3, find the dimension of Nulllspace(A) and Rowspace(A).

   (b) If the null space of an 8×7 matrix A is 5-dimensional, what is the dimension of the column space of A?

   (c) If the null space of an 8×5 matrix A is 3-dimensional, what is the dimension of the row space of A?

   (d) If A is a 7×5 matrix, what is the largest possible rank of A?

   (e) If A is a 5×7 matrix, what is the largest possible rank of A?