In all homework problems, it is not sufficient to show only the answers. *You must show your work.* These exercises are based on Chapter Three.I and Three.II from the text.

1. Let $f : \mathcal{P}_1 \to \mathbb{R}^2$ be defined by
$$f(a+bx) = \begin{bmatrix} a+b \\ a-b \end{bmatrix}.$$
   (a) Find the image of each of the following elements of the domain:
   (i) $1-x$ (ii) $-1+3x$ (iii) $4+4x$
   (b) For each of the following vectors $\begin{bmatrix} c \\ d \end{bmatrix} \in \mathbb{R}^2$, find a vector $a+bx \in \mathcal{P}_1$ with $f(a+bx) = \begin{bmatrix} c \\ d \end{bmatrix}$: (i) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$
   (c) Show that f is an isomorphism.

2. Neither of the following functions $f : \mathbb{R}^2 \to \mathbb{R}^2$ is an isomorphism. For each function identify a property in the definition of isomorphism that fails, and verify that that property fails.
   (a) $f(\begin{bmatrix} a \\ b \end{bmatrix}) = \begin{bmatrix} a+b \\ ab \end{bmatrix}$. (b) $f(\begin{bmatrix} a \\ b \end{bmatrix}) = \begin{bmatrix} a-b \\ 2a-2b \end{bmatrix}$.

3. For which n is the space isomorphic to $\mathbb{R}^n$?
   (a) $\mathcal{P}_3$ (b) $\mathcal{M}_{2\times 3}$ (c) The nullspace of $A = \begin{bmatrix} 1 & 2 & 0 & -1 & 4 \\ 0 & 0 & 1 & 3 & -2 \end{bmatrix}$

4. Verify that each map is a homomorphism.
   (a) $f : \mathbb{R}^2 \to \mathcal{P}_2$ given by $f(\begin{bmatrix} a \\ b \end{bmatrix}) = (a+b)x + (a-b)x^2$.
   (b) $g : \mathbb{R}^2 \to \mathbb{R}^3$ given by $f(\begin{bmatrix} a \\ b \end{bmatrix}) = \begin{bmatrix} a-b \\ 0 \\ a+b \end{bmatrix}$.

5. Find the (i) range space, (ii) rank, (iii) null space, and (iv) nullity for each of the following homomorphisms.
   (a) $f : \mathbb{R}^2 \to \mathcal{P}_3$ given by $f(\begin{bmatrix} a \\ b \end{bmatrix}) = (a+b) + (a+b)x + (a+b)x^2$.
   (b) $g : \mathcal{M}_{2\times 2} \to \mathbb{R}$ given by $f(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = b - c$.