In all homework problems, it is not sufficient to show only the answers. *You must show your work.* These exercises are based on Chapter Three.I and Three.II from the text.

1. Assume that each matrix represents a map $h\colon \mathbb{R}^n \to \mathbb{R}^m$ with respect to the standard bases. In each case, (i) state m and n (ii) find $\mathscr{R}(h)$ (range space of h) and rank(h) (iii) find $\mathscr{N}(h)$ (null space of h) and nullity(h), and (iv) state whether the map is onto and whether it is one-to-one.

   (a) $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 3 \\ 2 & 3 & 4 \\ -2 & -1 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$

2. Verify that the map $h\colon \mathbb{R}^n \to \mathbb{R}^m$ represented by this matrix with respect to the standard bases
$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 1 & 1 \\ 7 & 2 & 1 \end{bmatrix}$$
is an isomorphism.

3. Let the homomorphism $h : \mathbb{R}^3 \to \mathcal{P}_2$ be given by
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \mapsto (a+b)x^2 + (2a+2b)x + c.$$
For the bases $\mathcal{B} = \langle \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rangle$ of $\mathbb{R}^3$ and $\mathcal{C} = \langle 1+x,\, 1-x,\, x^2 \rangle$ of $\mathcal{P}_2$, find the matrix representation $\mathrm{Rep}_{\mathcal{B},\mathcal{C}}(h)$ of h with respect to the bases $\mathcal{B}$ and $\mathcal{C}$.

4. Let $h : \mathbb{R}^3 \to \mathcal{P}_2$ be the homomorphism represented with respect to the bases $\mathcal{E}_3$ of $\mathbb{R}^3$ and $\mathcal{C} = \langle 1,\, 1+x^2,\, x \rangle$ by the matrix
$$H = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}.$$

   (a) Find $h(\vec{v})$ for $\vec{v} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$.

   (b) Find $h(\vec{w})$ for the general vector $\vec{w} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

   (c) Determine if $1 - 3x + x^2$ and $3 - x + x^2$ are in the range of h.