

In all homework problems, it is not sufficient to show only the answers. *You must show your work.* These exercises are based on Chapter Four.I and Four.III from the text.

1. Compute the determinant of each of the following matrices. Show your work.

$$(a) \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad (b) \begin{bmatrix} 2 & 0 & 3 & 2 \\ 5 & 1 & 2 & 4 \\ 3 & 0 & 1 & 2 \\ 5 & 3 & 2 & 1 \end{bmatrix} \quad (c) \begin{bmatrix} 2 & 1 & 1 & 0 & 1 \\ 2 & 1 & 2 & -1 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 1 & 0 & 3 & 1 & 1 \\ 2 & 1 & 1 & 2 & 1 \end{bmatrix}$$

2. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. If $\det A = 7$, compute the determinant of each of the following matrices.

$$(a) \begin{bmatrix} a & b & d \\ 2d & 2e & 3f \\ 5g & 5h & 5i \end{bmatrix} \quad (b) \begin{bmatrix} 3d & 3e & 3f \\ a & b & c \\ g & h & i \end{bmatrix} \quad (c) \begin{bmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{bmatrix}$$

3. Compute $\det B^5$ where $B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

4. Verify that $\det(AB) = \det A \det B$ for each of the following pairs of matrices.

$$(a) A = \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix}$$

5. Let A and B be 3×3 matrices, with $\det A = 4$ and $\det B = -3$. Use properties of determinants to compute:

$$(a) \det AB \quad (b) \det 5A \quad (c) \det B^{-1} \quad (d) \det A^3 \quad (e) \det BAB^{-2}$$

6. For each of the following matrices, determine all of the values of x for which the matrix is not invertible.

$$(a) A = \begin{bmatrix} 2-x & 1 \\ 4 & 2-x \end{bmatrix} \quad (b) B = \begin{bmatrix} 4-x & -4 & -4 \\ 2 & -2-x & -4 \\ 3 & -3 & -4-x \end{bmatrix}$$