Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. [**12 Points**] Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 3 \\ -2 & 1 \end{bmatrix}$. Compute each of the products AB and BA that makes sense. If a product does not make sense, state why it does not make sense.

2. [**12 Points**] Give an example of each of the following:
   (a) A 4×3 matrix that is *not* in row-echelon form.
   (b) A 3×4 matrix that is in row-echelon form, but *not* in reduced row-echelon form.
   (c) A 4×4 matrix that is in reduced row-echelon form and has exactly 3 nonzero rows.
   (d) The 4×4 identity matrix I_4.

3. [**20 Points**] Find all solutions of the following system of linear equations. Be sure to show all your steps!

$$\begin{array}{rcrcrcrcl} x_1 & + & 2x_2 & + & 2x_3 & + & 6x_4 & = & 1 \\ x_1 & - & x_2 & + & 2x_3 & - & 6x_4 & = & 4 \\ x_1 & + & x_2 & + & 2x_3 & + & 2x_4 & = & 2 \end{array}$$

4. [**20 Points**] Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$.
   (a) Compute A^{-1}.
   (b) Using your answer to part (a), solve the system of equations

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & + & 3x_3 & = & 1 \\ x_1 & + & x_2 & + & 2x_3 & = & 0 \\ x_1 & + & x_2 & + & x_3 & = & 1 \end{array}$$

5. [**14 Points**] Find conditions that b_1, b_2, and b_3 must satisfy for the following system to be consistent:

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & - & 3x_3 & = & b_1 \\ 2x_1 & + & 3x_2 & + & x_3 & = & b_2 \\ 5x_1 & + & 9x_2 & - & 8x_3 & = & b_3 \end{array}$$

6. [**12 Points**] Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 7 \\ 2 & 2 & 7 \end{bmatrix}$.

   (a) Find elementary matrices E_1, E_2, and E_3 so that $E_1 E_2 E_3 A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$.

   (b) Using your answer to part (a), compute $\det A$.

7. [**10 Points**] Let A be an $n \times n$ invertible matrix. Determine whether each of the following statements is True (**T**) or False (**F**).

   (a) $Ax = 0$ has only the solution $x = 0$.

   (b) $A^{-1}x = 0$ has infinitely many solutions.

   (c) The reduced row-echelon form of A has a row of zeros.

   (d) $Ax = b$ has a unique solution for every $n \times 1$ matrix b.

   (e) $\det A \neq 0$.