Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. [**12 Points**] Which of the following subsets of the set of 2×2 matrices M_{22} are subspaces of M_{22}? Give a (brief) reason in each case.

   (a) S_1 is the set of all matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$.

   (b) S_2 is the set of all matrices of the form $\begin{bmatrix} a & b \\ 2 & c \end{bmatrix}$.

2. [**24 Points**] Let $A = \begin{bmatrix} 2 & -4 & 1 & 1 & 0 \\ -1 & 2 & -1 & 1 & 6 \\ 2 & -4 & 0 & 4 & 12 \\ 1 & -2 & 3 & 5 & 30 \end{bmatrix}$ and let $R = \begin{bmatrix} 1 & -2 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Then R is the reduced row-echelon matrix associated to A.

   (a) Compute a basis for the row space $\text{Row}(A)$ of A.

   (b) Compute a basis for the column space $\text{Col}(A)$ of A.

   (c) Compute a basis for the nullspace $\text{Null}(A)$ of A.

   (d) What is the rank of A?

3. [**20 Points**]

   (a) Show that the vectors $\mathbf{v}_1 = (1, 2, 3, 4)$, $\mathbf{v}_2 = (0, 1, 0, -1)$, and $\mathbf{v}_3 = (1, 3, 3, 3)$. form a linearly dependent set in $\mathbb{R}^4$.

   (b) Express each of the vectors as a linear combination of the others.

4. [**20 Points**] Let $S = \{\mathbf{v}_1, \mathbf{v}_2\}$ be the basis of $\mathbb{R}^2$ with $\mathbf{v}_1 = (1, 2)$ and $\mathbf{v}_2 = (1, 3)$.

   (a) If $\mathbf{v} = (2, 4)$ compute the coordinate vector $(\mathbf{v})_S$.

   (b) If $(\mathbf{w})_S = (3, -2)$, then find $\mathbf{w}$.

5. [**12 Points**] Let A be a 5×7 matrix. and assume that the dimension of the nullspace of A is 4.

   (a) What is the dimension of the column space of A?

   (b) What is the rank of A?

   (c) What is the dimension of the nullspace of A^T (Recall that A^T is the transpose of A.)

6. [**12 Points**] Complete the following definitions:

   (a) A set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of vectors in a vector space V is *linearly dependent* if ...

   (b) A *basis* of a vector space V is a set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ of vectors such that ...

   (c) A vector $\mathbf{w}$ is a *linear combination* of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ if ...