Math 2085 Section 1
Exam I

Instructions. Work on your own paper and show all relevant work. Be sure to read each problem carefully and do exactly what is requested, no more and no less. For convenience in comparing with the class roll, please *print* your name and student number in the spaces provided.

Name:

Student Number:

(12) 1. Let A be an $n \times n$ matrix. Give two different statements which are equivalent to the statement "A is invertible."

(14) 2. Let $A = \begin{bmatrix} 3 & 2 & 7 & 5 \\ 2 & 1 & 3 & 6 \\ 1 & 2 & 9 & 3 \end{bmatrix}$.

(a) Compute the reduced row-echelon matrix R which is row equivalent to A.
(b) Write the system of linear equations for which A is the augmented matrix. (Do **not** solve this system.)
(c) Write the system of homogeneous linear equations for which A is the coefficient matrix. (Do **not** solve this system.)

(12) 3. In each part suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given reduced row-echelon form. Solve the system.

(a) $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(12) 4. Now consider each of the matrices in Exercise 3 as the coefficient matrix of a homogeneous linear system, and solve the system.

(14) 5. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$.

(a) Compute A^{-1}.

(b) Using your answer to part (a), solve the matrix equation $AX = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$.

(12) 6. Find the conditions that b_1, b_2, and b_3 must satisfy for the following system to be consistent:

$$\begin{aligned} x_1 + x_2 - x_3 &= b_1 \\ -2x_1 + x_2 + 3x_3 &= b_2 \\ x_1 - 2x_2 - 2x_3 &= b_3 \end{aligned}$$

(12) 7. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 5 & 3 \\ 0 & 0 & -11 \end{bmatrix}$. Assuming that $\det A = 5$, compute the following determinants:

(a) $\det(3A)$ (b) $\det(A^{-1})$ (c) $\det(BA^{-1})$ (d) $\det \begin{bmatrix} a & b & c \\ d-2a & e-2b & f-2c \\ 3g & 3h & 3i \end{bmatrix}$

(12) 8. Let $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find elementary matrices E, F, and G such that $B = EFGA$.