13. $\frac{1}{4}(m^{12} + 2m^6 + m^8)$

14. $\frac{1}{4}(m^{24} + 3m^{12})$

15. $\frac{1}{4}(m^{35} + m^{18} + m^{20} + m^{21})$

16.

$$k = \begin{cases} \frac{1}{4}(m^{pq} + m^{(pq+1)/2} + m^{(pq+p)/2} + m^{(pq+q)/2}) & \text{if } p, q \text{ are both odd} \\ \frac{1}{4}(m^{pq} + 3m^{pq/2}) & \text{if } p, q \text{ are both even} \\ \frac{1}{4}(m^{pq} + 2m^{pq/2} + m^{(pq+p)/2}) & \text{if } p \text{ is even and } q \text{ is odd} \\ \frac{1}{4}(m^{pq} + 2m^{pq/2} + m^{(pq+q)/2}) & \text{if } p \text{ is odd and } q \text{ is even} \end{cases}$$

17. $\frac{1}{12}(m^4 + 11m^2)$

18. $\frac{1}{24}(m^6 + 8m^2 + 12m^3 + 3m^2)$

EXERCISES 5.6

1. $\frac{1}{24}\left(y_1^4 + 6y_1^2 y_2 + 3y_2^2 + 8y_1 y_3 + 6y_4\right)$

2. $\frac{1}{120}\left(y_1^5 + 10y_1^3 y_2 + 20y_1^2 y_3 + 15y_1 y_2^2 + 20y_2 y_3 + 30y_1 y_4 + 24y_5\right)$

3. $\frac{1}{12}\left(y_1^4 + 3y_2^2 + 8y_1 y_3\right)$

4. $\frac{1}{8}\left(y_1^4 + 2y_1^2 y_2 + 3y_2^2 + 2y_4\right)$

5. $\frac{1}{4}\left(y_1^4 + 3y_2^2\right)$

6. $\frac{1}{12}\left(y_1^6 + 3y_1^2 y_2^2 + 4y_2^3 + 2y_3^2 + 2y_6\right)$

7. $\frac{1}{2n}\left(\sum_{r|n} \varphi(r) y_r^{n/r} + n y_1 y_2^{(n-1)/2}\right)$ if n is odd;
$\frac{1}{2n}\left(\sum_{r|n} \varphi(r) y_r^{n/r} + \frac{n}{2}\left(y_1 y_2^{(n-1)/2} + y_2^{n/2}\right)\right)$ if n is even

EXERCISES 5.2

1. D_3, D_2, D_4, D_2, D_2
2. C_1 : F, G, J, L, P, Q, R; C_2 : N, S, Z; D_1 : A, B, C, D, E, K, M, T, U, V, W, X, Y; D_2 : H, I, O
3. D_2, D_1, D_2
4. D_1, D_2, D_1, D_4, D_3
7. See the table given in the text.
8. The rotational symmetries of the cube correspond to the permutations of the four diagonals.
9. Suppose $a^i \in Z(D_n)$. Then $a^i b = ba^i = (a^{n-1})^i b = a^{-1}b$, so $a^{2i} = e$. Hence $2i = 0$ or n. Therefore $Z(D_n) = \{e\}$ or $\{e, a^{n/2}\}$, according to whether n is odd or even.

EXERCISES 5.5

1. $\frac{1}{8}(m^4 + 2m + 3m^2 + 2m^3)$
2. $\frac{1}{10}(m^5 + 4m + 5m^3)$
3. $\frac{1}{4}(m^4 + 3m^2)$
4. $\frac{1}{2p}(n^p + (p-1)n + pn^{(p+1)/2})$
5. $\frac{1}{12}(3^6 + 2 \cdot 3 + 2 \cdot 3^2 + 4 \cdot 3^3 + 3 \cdot 3^4) = 92$

8. $\frac{1}{8}(3^{12} + 2 \cdot 3^3 + 3 \cdot 3^6 + 2 \cdot 3^7) = 67,257$
9. $\frac{1}{2n}\left(\sum_{r|n} \varphi(r) m^{qn/r} + N\right)$, where

$$N = \begin{cases} nm^{qn/2} & \text{if } q \text{ is even} \\ nm^{(qn+1)/2} & \text{if } q \text{ is odd and } n \text{ is odd} \\ \frac{n}{2}(m^{qn/2} + m^{(qn+2)/2}) & \text{if } q \text{ is odd and } n \text{ is even} \end{cases}$$

10. $\frac{1}{6}(4^3 + 2 \cdot 4 + 3 \cdot 4^2) - 4 = 16$
11. $\frac{1}{8}(4^4 + 2 \cdot 4 + 3 \cdot 4^2 + 2 \cdot 4^3) - 4 = 51$