Cyclic Group Exercises

1. Find all generators of the cyclic group \( G = \langle g \rangle \) if:
   
   (a) \( o(g) = 5 \)  
   (b) \( o(g) = 10 \)  
   (c) \( |G| = 16 \)  
   (d) \( |G| = 20 \)

2. Find all generators of:
   
   (a) \( \mathbb{Z}_5 \)  
   (b) \( \mathbb{Z}_{10} \)  
   (c) \( \mathbb{Z}_{16} \)  
   (d) \( \mathbb{Z}_{20} \)

3. In each case determine whether \( G \) is cyclic.
   
   (a) \( G = \mathbb{Z}_7^* \)  
   (b) \( G = \mathbb{Z}_{12}^* \)  
   (c) \( G = \mathbb{Z}_{16}^* \)  
   (d) \( G = \mathbb{Z}_{11}^* \)

4. Let \( o(g) = 20 \) in a group \( G \). Compute:
   
   (a) \( o(g^2) \)  
   (b) \( o(g^5) \)  
   (c) \( o(g^5) \)  
   (d) \( o(g^3) \)

5. In each case find all the subgroups of \( G = \langle g \rangle \) and draw the lattice diagram.
   
   (a) \( o(g) = 8 \)  
   (b) \( o(g) = 10 \)  
   (c) \( o(g) = 18 \)  
   (d) \( o(g) = p^3 \), where \( p \) is prime.
   (e) \( o(g) = pq \), where \( p \) and \( q \) are distinct primes.
   (f) \( o(g) = p^2 q \), where \( p \) and \( q \) are distinct primes.

6. In each case, find the subgroup \( H = \langle x, y \rangle \) of \( G \).
   
   (a) \( G = \langle a \rangle \) is cyclic, \( x = a^4 \), \( y = a^3 \).
   (b) \( G = \langle a \rangle \) is cyclic, \( x = a^6 \), \( y = a^8 \).
   (c) \( G = \langle a \rangle \) is cyclic, \( x = a^m \), \( y = a^k \), \( \gcd(m, k) = d \).
   (d) \( G = S(3) \), \( x = (1, 2) \), \( y = (2, 3) \).