Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. **[18 Points]**
   (a) Calculate the greatest common divisor $d = (378, 490)$ by the Euclidean algorithm, and write d in the form $378 \cdot s + 490 \cdot t$ for some integers s and t.
   (b) Calculate $[378, 490]$.
   (c) Describe all of the integers n that can be written in the form $n = 378 \cdot u + 490 \cdot v$ for some integers u and v. That is, identify the set S in the equation $378\mathbb{Z} + 490\mathbb{Z} = S$.

2. **[20 Points]** This exercise makes use of the following equation:

$$1 = 7 \cdot 103 - 48 \cdot 15.$$

   Using this equation (i.e., *do not* use the Euclidean algorithm to recreate it), answer the following questions.
   (a) Compute the multiplicative inverse of $[15]_{103}$ in $\mathbb{Z}_{103}$. Express your answer in the standard form $[b]_{103}$ where $0 \le b < 103$.
   (b) Solve the congruence equation $15x \equiv 8 \pmod{103}$. Express your answer in the standard form $x \equiv b \pmod{103}$ where $0 \le b < 103$.
   (c) Solve the system of simultaneous linear congruences:

$$\begin{aligned} x &\equiv 8 \pmod{103} \\ x &\equiv 3 \pmod{48}. \end{aligned}$$

   Give your answer in for form $x \equiv r \pmod{m}$ where r is the smallest possible *positive* integer and m is determined from the problem.

3. **[20 Points]** This problem concerns arithmetic modulo 21. All answers should be expressed in the standard form $[a]_{21}$ with a an integer satisfying $0 \le a < 21$.
   (a) Compute $[9]_{21} + [16]_{21}$.
   (b) Compute $[9]_{21}[16]_{21}$.
   (c) Compute $[5]_{21}^{-1}$.
   (d) Find a nonzero $[b]_{21}$ such that $[12]_{21}[b]_{21} = [0]_{21}$.
   (e) List the invertible elements of $\mathbb{Z}_{21}$.
   (f) List the zero divisors of $\mathbb{Z}_{21}$.

4. **[18 Points]**
   (a) If a and b are integers, write the definition of the statement "a divides b". Be sure to write in a complete sentence.
   (b) If a, b, and c are integers such that $a|b$ and $a|(b + c)$, then prove, directly from the definition of divides (which you have conveniently provided in Part (a)), that $a|c$.

5. [**12 Points**] Let a, b, c be integers, where $a \neq 0$ or $b \neq 0$.

   (a) Fill in the box with an appropriate statement about a and b to provide a true result:

   If $b|ac$ *and* $\boxed{}$, *then* $b|c$.

   (b) Provide an example of integers a, b, c for which $b|ac$, but $b \nmid c$. Naturally, the condition you listed in part (a) will not be satisfied.

6. [**12 Points**] For each of the following relations on the real numbers $\mathbb{R}$, determine which of the properties (i) reflexive, (ii) symmetric, and (iii) transitive hold.

   (a) For $a, b \in \mathbb{R}$, define $a \sim b$ if $a \leq b$.

   (b) For $a, b \in \mathbb{R}$, define $a \sim b$ if $|a - b| \leq 1$.

   Are either of these relations an equivalence relation?