Instructions. Answer each of the questions on your own paper, and be sure to show your work so that partial credit can be adequately assessed. Put your name on each page of your paper.

1. [**16 Points**] Calculate the greatest common divisor $d = \gcd(525, 231)$ by the Euclidean algorithm, and write d in the form $525 \cdot s + 231 \cdot t$ for some integers s and t.

2. [**18 Points**] Use induction to prove that the equation

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

is valid for all integers $n \geq 1$. That is, the sum of the first n odd integers is equal to n^2.

3. [**18 Points**] Let $A \xrightarrow{\alpha} B \xrightarrow{\beta} A$ satisfy $\beta\alpha = 1_A$.
   (a) Show that α is one-to-one.
   (b) Give an example with $A = \mathbb{N} = B$ such that α does not have an inverse.

4. [**16 Points**] Find all solutions to the system of simultaneous linear congruences:

$$\begin{aligned} x &\equiv 9 \pmod{16} \\ x &\equiv 2 \pmod{33}. \end{aligned}$$

5. [**16 Points**] In $\mathbb{Z}_{35}$ find the inverse of $\overline{13}$ and use it to solve the equation $\overline{13}x = \overline{9}$.

6. [**16 Points**] Let $\sigma = (1\ 2\ 5\ 4)(2\ 4\ 6)(3\ 6\ 4) \in S_6$.
   (a) Write σ in two-row notation, i.e., complete the second row of the following matrix:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \sigma(1) & \sigma(2) & \sigma(3) & \sigma(4) & \sigma(5) & \sigma(6) \end{pmatrix}.$$

   (b) Write σ as a product of *disjoint* cycles.