Instructions. Answer each of the questions on your own paper, and be sure to show your work, including giving reasons, so that partial credit can be adequately assessed. Put your name on each page of your paper. There is a total possible of 75 points.

1. [**20 Points**] Let $\sigma \in S_9$ be the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 4 & 7 & 6 & 9 & 8 & 3 & 2 & 1 \end{pmatrix}.$$

   (a) Express σ as a product of disjoint cycles. Is σ even or odd?

   (b) Find the order of σ.

   (c) Express σ^{-1} as a product of disjoint cycles.

   (d) Let $\tau = \begin{pmatrix} 1 & 2 \end{pmatrix}$. Compute $\tau\sigma$. Give your answer as a product of disjoint cycles.

2. [**15 Points**] Let G be a finite group with $|G| = 10$.

   (a) What are the possible orders of elements of G?

   (b) If H is a subgroup of G and $H \neq G$, prove that H is cyclic.

   (c) Give an example of a nonabelian group of order 10. (*Hint.* What is the order of the dihedral group D_n?)

3. [**15 Points**] Let $G = U(15)$ be the group of integers modulo 15 that have a multiplicative inverse. Thus, $G = U(15) = \{1, 2, 4, 7, 8, 11, 13, 14\}$. Let $H = \langle 4 \rangle$ be the cyclic subgroup of G generated by 4.

   (a) Find $|H|$ and $[G : H]$, the number of cosets of H in G.

   (b) Give a list of all of the *distinct* cosets of H in G.

   (c) Is the factor group G/H a cyclic group? If so, find a generator. If not, show that it is not cyclic.

4. [**15 Points**]

   (a) State Lagrange's Theorem. Be sure to include any hypotheses.

   (b) What is the relationship between the order of an element $g \in G$ and the order of the group G?

   (c) If $g \in G$ is an element such that $g \neq 1$ and $g^{20} = g^{30} = 1$, then what can you conclude about $|g|$, the order of g?

5. [**10 Points**] Let $\mathbb{R}$ denote the group of real numbers under addition and $\mathbb{R}^*$ denote the group of non-zero real numbers under multiplication. Determine whether each of the given mappings is a homomorphism. Justify your answer briefly.

   (a) Define $\phi : \mathbb{R} \to \mathbb{R}$ by $\phi(x) = 3x$ for all $x \in \mathbb{R}$.

   (b) Define $\psi : \mathbb{R}^* \to \mathbb{R}^*$ by $\phi(x) = 3x$ for all $x \in \mathbb{R}^*$.