Do the following exercises from Judson:
Chapter 5, Section 5.3: 7, 14, 30, 34
Chapter 6, Section 6.4: 16, 17

Exercises not from the text:

1. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 8 & 9 & 5 & 2 & 1 & 6 & 4 & 7 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 8 & 6 & 9 & 4 & 7 & 3 & 1 & 5 \end{pmatrix}$. For each of the permutations σ, τ, σ^2, $\sigma\tau$, and $\tau\sigma$, determine the following.
   (a) Find the disjoint cycle factorization of each permutation.
   (b) Compute the order of each permutation.
   (c) Determine if each permutation is even or odd.

2. Let $G = D_4 = \{1, r, r^2, r^3, s, sr, sr^2, sr^3\}$ where $r^4 = 1$, $s^2 = 1$, $srs = r^{-1}$. (See Theorem 5.23) A representation as permutations is $r = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$ and $s = \begin{pmatrix} 2 & 4 \end{pmatrix}$. Let $H = \langle r^2 \rangle$ and $K = \langle s \rangle$. Determine all of the left and right cosets of the subgroups H and K of G.

3. Let G be a group and let $g \in G$ be an element. Suppose $|G| = 40$, $g^8 \neq 1$, and $g^{20} \neq 1$. Show that $G = \langle g \rangle$.

4. Suppose a group G has subgroups of orders 45 and 75. If $|G| < 400$ determine $|G|$.