1. Calculate the number of elements of order 2 in each of the abelian groups $\mathbb{Z}_{16}$, $\mathbb{Z}_8 \times \mathbb{Z}_2$, $\mathbb{Z}_4 \times \mathbb{Z}_4$, and $\mathbb{Z}_4 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. Do the same for elements of order 4.

2. Prove that every abelian group of order 45 has an element of order 15. Does every abelian group of order 45 have an element of order 9?

3. The symmetry group of a nonsquare rectangle is an abelian group of order 4. Is it isomorphic to $\mathbb{Z}_4$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$?

4. The set $G = \{1,\ 9,\ 16,\ 22,\ 29,\ 53,\ 74,\ 79,\ 81\}$ is a group under multiplication modulo 91. Determine the isomorphism class of this group.

5. Let $G = \{1,\ 7,\ 17,\ 23,\ 49,\ 55,\ 65,\ 71\}$ under multiplication modulo 96. Express G as an external and an internal direct product of cyclic groups.

6. The set $G = \{1,\ 4,\ 11,\ 14,\ 16,\ 19,\ 26,\ 29,\ 31,\ 34,\ 41,\ 44\}$ is a group under multiplication modulo 45. Write G as an external and an internal direct product of cyclic groups of prime-power order.