Do the following exercises from Beachy-Blair:

Page 234: 2(a), (b); 3(b), (f); 7;
Page 249: 7

Supplemental Exercises (i.e., not from the text).

1. Let $R$ be a commutative ring with $a, b \in R$.
   
   (a) If $ab$ is a unit, show that both $a$ and $b$ are units.
   
   (b) If $ab$ is a divisor of zero, then either $a$ or $b$ is a divisor of zero.

2. (a) If $F$ is a field and $f(x) \in F[x]$ is a nonzero polynomial, show that $f(x)$ has a multiplicative inverse if and only if the degree of $f(x)$ is 0.
   
   *Hint:* Proposition 4.1.5 will be useful.

   (b) If $R = \mathbb{Z}_4$, show that $f(x) = 2x + 1$ has a multiplicative inverse in $R[x]$.

3. What is the characteristic of the ring $\mathbb{Z}_m \oplus \mathbb{Z}_n$?

4. Let $\phi : \mathbb{Z}[i] \to \mathbb{Z}_5$ be defined by $\phi(n + mi) = [n + 2m]_5$.
   
   (a) Show that $\phi$ is a ring homomorphism.
   
   (b) What is $\text{ker}(\phi)$?