Math 7200
Midterm Examination
October 20, 1999

1. Let $G = \langle a \rangle$ be a cyclic group of order 360.
   (a) Let $b = a^{12}$, $c = a^{30}$, $H = \langle b \rangle$, and $K = \langle c \rangle$. What is $|H|$ and $|K|$? Prove that your answers are correct.
   (b) Determine generators for each of the subgroups $H \cap K$ and HK, and determine the orders of each of these subgroups.
   (c) Is G the internal direct product of H and K? Justify your answer.
2. Let V be a finite dimensional vector space over a field F and let U and W be subspaces. Prove that if $\dim U + \dim W > \dim V$, then $U \cap W \neq \{0\}$.
3. Let V be a vector space of dimension 3 over $\mathbf{C}$, let $\mathcal{B} = (v_1, v_2, v_3)$ be an ordered basis for V and let $T \in L(V)$ be the linear operator defined by

$$T(v_1) = 3v_2, \quad T(v_2) = -0, \quad T(v_3) = 5v_1 - 4v_2.$$

   (a) Compute the matrix $[T]_{\mathcal{B}}$.
   (b) Show that T is nilpotent, i.e., $T^k = 0$ for some k.
   (c) Determine a basis $\mathcal{C} = (w_1, w_2, w_3)$ of V so that the matrix $[T]_{\mathcal{C}}$ is in Jordan canonical form.
4. Let T be a linear operator on the complex vector space V. Assume that $\dim V = 6$ and the minimal polynomial of T is

$$\mu_T(X) = (X-2)^2(X-3)^2.$$

   Find all Jordan forms for T consistent with these data.