1. The group $\mathbb{Z}_n^*$ is the group of congruence classes $[a]_n$ of integers modulo n for which a and n are relatively prime. The group operation is multiplication of congruence classes. Write out multiplication tables for each of the groups $\mathbb{Z}_9^*$ and $\mathbb{Z}_{15}^*$.

2. On the set $G = \mathbb{Q}^*$ of nonzero rational numbers, define a new multiplication by $a * b = \frac{ab}{2}$, for all a and b in G. Prove that G is a group under the operation $*$. Recall that this means that you must prove that $*$ is an associative binary operation on G, that there is an identity element $e \in G$, and for each $a \in G$, there is a $b \in G$ for which $ab = ba = e$. Note that you must explicitly identify e, verify that it is an identity, and for each $a \in G$ you must explicitly identify the inverse element b and verify that it is in fact the inverse of a.

3. Let G be a group and let a and b be any elements of G. Show that $(ab)^2 = a^2b^2$ if and only if $ab = ba$. Explain why this shows that G is abelian if and only if the squaring map $\varphi : G \to G$ defined by $\varphi(a) = a^2$ is a group homomorphism.

4. Let G be a group and let a and b be any elements of G. Give a careful argument using induction to show that $(aba^{-1})^n = ab^na^{-1}$ for all positive integers n.

Do the following exercises from Chapter 1 of the text (pages 45 – 48): 7, 11