From the text (pages 45 – 48): 30, 31, 32.

1. Let σ and τ be the following permutations in S_{10}:

$$\begin{aligned} \sigma &= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 6 & 7 & 8 & 1 & 10 & 5 & 2 & 9 & 4 \end{pmatrix} \\ \tau &= (8\ 9\ 10)(2\ 3\ 6\ 7)(5\ 4\ 2)(9\ 3\ 6\ 8)(1\ 4\ 7) \end{aligned}$$

   Find the disjoint cycle decomposition, order, and parity for each of the following permutations: σ, τ, σ^2, $\sigma\tau$, $\tau\sigma$, $\sigma^2\tau$, and $\sigma\tau\sigma^{-1}$.

2. Let σ and τ be the following permutations in S_7:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 2 & 1 \end{pmatrix} \text{ and } \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 1 & 7 & 6 & 2 & 3 \end{pmatrix}.$$

   Are σ and τ conjugate in S_7? If so, find $\gamma \in S_7$ such that $\gamma\sigma\gamma^{-1} = \tau$.

3. (a) If $1 \le r \le n$, show that there are

$$\frac{1}{r}[n(n-1)\cdots(n-r+1)]$$

   r-cycles in S_n.

   (b) The formula in part (a) can be used to count the number of permutations having any given cycle structure if one is careful about factorizations having several cycles of the same length. For example, the number of permutations in S_4 of the form $(a\ b)(c\ d)$ is

$$\frac{1}{2}\left[\frac{1}{2}(4 \times 3)\right] \times \left[\frac{1}{2}(2 \times 1)\right] = 3,$$

   the extra factor $\frac{1}{2}$ occurring so that we do not count $(a\ b)(c\ d) = (c\ d)(a\ b)$ twice. Using these procedures we arrive at the following table delineating the number of permutations of each cycle type in S_4:

Cycle Structure	Number
(a)	1
$(a\ b)$	6
$(a\ b\ c)$	8
$(a\ b\ c\ d)$	6
$(a\ b)(c\ d)$	3

   Notice that the sum of the second column is $24 = |S_4|$. Prepare a table like the one above listing the number of permutations of each possible cycle type in S_5.

4. (a) How many permutations in S_5 commute with $(1\ 2\ 3)$, and how many *even* permutations commute with $(1\ 2\ 3)$? [*Hint:* Proposition (2.27) may be useful.]

   (b) Same question for $(1\ 2)(3\ 4)$.