From the text (pages 45 – 48): 49, 50, 52.

1. Let G act on a set X. Assume that $y = gx$ where $g \in G$ and x, $y \in X$. Prove that the stabilizers $G(x)$ and $G(y)$ are conjugate subgroups of G.

2. Let G be a p-group with $|G| = p^n$. Show that any subgroup of G of order p^{n-1} must be normal in G.

3. Suppose that $n \geq 3$. Is S_n isomorphic to a direct product $A_n \times G$ where G is a group of order 2? Naturally, a proof of your claim is required.

4. List all 3-Sylow subgroups of A_4 and list all 3-Sylow subgroups of S_4.

5. List all 2-Sylow subgroups of S_4 and find elements of S_4 which conjugate one of these into each of the others.