1. Give an example of fields $F \subset E \subset K$ such that K is a root extension of F but E is not a root extension of F. (Hint: Look at $K = \mathbb{Q}(\zeta)$ where ζ is a primitive 7^{th} root of unity.)

2. Let $F = \mathbb{Q}$, $E = \mathbb{Q}(\sqrt{3})$, $K = \mathbb{Q}(\sqrt{\sqrt{3}+1})$. Show that E/F and K/E are Galois extensions, but that K/F is not Galois. Find the minimal polynomial of $\sqrt{\sqrt{3}+1}$ and find it's Galois group.

3. Find a root extension of $\mathbb{Q}$ containing the splitting fields of each of the following polynomials.

   (a) $x^4 + 1$

   (b) $x^4 + 3x^2 + 1$

   (c) $x^5 + 4x^3 + x$

   (d) $(x^3 - 2)(x^7 - 5)$

4. Give an example of a polynomial in $\mathbb{Q}[x]$ which is solvable by radicals, but whose splitting field is not a root extension of $\mathbb{Q}$.

5. For r a positive integer, define $f_r(x) \in \mathbb{Q}[x]$ by

$$f_r(x) = (x^2 + 4)x(x^2 - 4)(x^2 - 16)\cdots(x^2 - 4r^2).$$

   (a) Give a (rough) sketch of the graph of $f_r(x)$.

   (b) Show that if k is an odd integer, then $|f_r(k)| \geq 5$.

   (c) Show that $g_r(x) = f_r(x) - 2$ is irreducible over $\mathbb{Q}$ and determine its Galois group when $2r + 3 = p$ is prime.