Do the following exercises from Adkins-Weintraub, Chapter 7:

2, 4, 5, 11, 17

Additional Exercises.

1. If R is a ring and P is an R-module, then P is **projective** if it satisfies the following condition:

   If M and N are R-modules, $g : M \to N$ is a surjective R-module homomorphism, and $f : P \to N$ is any R-module homomorphism, then there is an R-module homomorphism $h : P \to M$ with $f = g \circ h$.

   Prove that the following conditions on an R-module P are equivalent.

   (a) P is projective.

   (b) P is isomorphic to a direct summand of a free R-module.

   (c) If $f : M \to P$ is surjective, then there exists an R-module homomorphism $g : P \to M$ such that $f \circ g = \mathrm{id}_P$.

2. Let F be a field and let $R = F \times F$. Let $e = (1, 0) \in R$ and let $P = Re$. Show that P is a projective R-module, but that P is not a free R-module.

3. Show that if R is a semisimple ring, then so is $M_n(R)$.

4. Show that if R is a semisimple ring and I is any ideal, then R/I is also semisimple.