Abstracts

Akram Aldroubi: Union of subspace clustering, sparse approximations and dimensionality reduction

The subspace clustering problem proposes to find a union of subspaces that are closest to a given set of data. This is a generalization of the classical least squares, and It has many applications in computer science and engineering, such as face recognition and motion tracking in videos. However, the subspace clustering problem is highly non-linear and the data often lives in a very high dimensional space while the subspaces model consists of low dimensional vector spaces. In this talk we will set up the problem, relates some of the theoretical results about existence of solutions, algorithms for solving it, and how to reduce the problem to a more tractable low dimensional one.

Gaik Ambartsoumian: Exterior Problem of Acoustic Reflectivity Imaging

In various applications of acoustic reflection tomography the support of the image function lies outside of the data acquisition set, which is usually a closed curve in 2D or a closed surface in 3D. The problem of image reconstruction in this setup is highly unstable, yet necessary and important (e.g. in intravascular ultrasound). Although one can not expect stable reconstruction of the whole image, microlocal considerations show that certain image singularities can be reconstructed correctly. There are some uniqueness results and exact inversion formulas for a few restricted cases of this exterior problem, however no robust inversion algorithm has been developed so far to recover the "visible" singularities. The talk will discuss the known results and recent advances in this direction.

Gregory Backus: A Categorization of Mexican Free-Tailed Bat (Tadarida brasiliensis) Chirps

Male Mexican Free-tailed Bats (Tadarida brasiliensis) attract mates and defend territory using multi-phrase songs that follow a structured set of rules. A subjective view of their spectrograms shows similarity and dissimilarity between the chirps (a syllable within the song) of different males. We developed an algorithm to characterize the shapes of these chirps. In order to compress data, the discrete Fourier transform and a four level Daubechies 2 wavelet decomposition allowed us to divide the chirps into segments based on frequency information after we cut the chirps into time-based segments. After calculating these segments' energies, we compressed the large data vectors into a single data point using multidimensional scaling. When comparing our first dimension of this multidimensional scaling, wavelet analysis gave us categorizations that most closely resembled our subjective categorizations.

B. A. Bailey: Multivariate Polynomial Interpolation and Sampling in Paley-Wiener Spaces

In this presentation, an equivalence between existence of particular exponential Riesz bases for multivariate bandlimited functions and existence of certain polynomial interpolants for these bandlimited functions is given. For certain classes of unequally spaced data nodes and corresponding ℓ_2 data, the existence of these polynomial interpolants allows for a simple recovery formula for multivariate bandlimited functions which demonstrates L_2 and uniform convergence on $\mathbb{R}^d$. A simpler computational version of this recovery formula is also given, at the cost of replacing L_2 and uniform convergence on $\mathbb{R}^d$ with L_2 and uniform convergence on increasingly large subsets of $\mathbb{R}^d$. As a special case, the polynomial interpolants of given ℓ_2 data converge in the same fashion to the multivariate bandlimited interpolant of that same data. Concrete examples of pertinant Riesz bases and unequally spaced data nodes are also given.

Cristina Balderrama: Generalized orthogonal polynomials and associated Markov semigroups.

A generalized polynomial is a central function, that is to say, a function with argument in the space of hermitian matrices H_n that only depends on the eigenvalues of the of the hermitian matrix in which it is being evaluated. The class of central functions on H_n is are intimately related to the class of symmetric functions on $\mathbb{R}$. We construct and study orthogonal basis of generalized polynomials associated to a family of orthogonal polynomials on $\mathbb{R}$, by means of a related family of symmetric orthogonal polynomials on $\mathbb{R}^n$.

Markov semigroups are closely related to Markov processes, a class of stochastic processes of great interest and many applications. As shown in [1], given a family of orthogonal polynomials on $\mathbb{R}$, it is possible to construct a Markov semigroup of operators having them as eigenfunctions, via the notion of Markov generator sequence. From the Markov semigroup associated to the orthogonal polynomials on $\mathbb{R}$ we construct semigroups of operators with eigenfunctions given by the related symmetric and generalized orthogonal polynomials. We study the infinitesimal generator of these semigoups and we give sufficient conditions for these semigroup to also be Markov.

This is a joint work with Piotr Graczyk and Wilfredo Urbina.

References

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Jeremy J. Becnel: A Support Theorem for Infinite Dimensional Gaussian Radon Transform

We show that as in finite dimensions, if a bounded, suitably continuous, function has zero Gaussian integral over all hyperplanes outside a closed bounded convex set then the function is zero outside this set. This is an infinite dimensional version of the well-known Helgason Support Theorem for Radon transforms in finite dimensions.

Jens Christensen: Sampling of band limited functions for Gelfand pairs

We present a generalization of the classical sampling theorem of band limited functions for the Fourier transform on $L^2(G/K)$ when (G, K) is a Gelfand pair.

Brad Currey (joint work with Azita Mayeli): Admissibility for representations of connected Lie groups

Let G be a locally compact, second countable group. A unitary representation π of G acting in a Hilbert space $\mathcal{H}$ is said to be admissible if there is $\psi \in \mathcal{H}$ such that the mapping

$$W_\psi : f \mapsto \langle f, \pi(\cdot)\psi \rangle$$

is an isometry of $\mathcal{H}$ into $L^2(G)$; in this case, W_ψ is a generalization of the continuous wavelet transform. We are interested in both necessary and sufficient conditions that a representation be admissible, and for admissible representations, explicit constructions for (generalized) continuous wavelets ψ. We present results for various Lie groups, including very recent joint work with Vignon Oussa.

Susanna Dann: Representation Theory and Paley-Wiener Theorems

$\mathbb{R}^n$ can be represented as a homogeneous space: $\mathbb{R}^n \simeq G/\mathrm{SO}(n)$, where G is the orientation preserving Euclidean motion group $\mathbb{R}^n \rtimes \mathrm{SO}(n)$. This realization comes with its own natural Fourier transform that is derived from the represenation theory of G. The representations of G that contribute to the decomposition of $L^2(\mathbb{R}^n)$ are parametrized by $\mathbb{R}^+$. We obtain Paley-Wiener type theorems which describe the image of smooth compactly supported scalar-valued and vector-valued functions under the Fourier transform with respect to the spectral parameter $\mathbb{R}^+$.

Suresh Eswarathasan: Microlocal analysis of backscattering for nested conormal potentials.

Consider the scattering problem for the perturbed wave equation

$$
\begin{aligned}
&(\Box + q(x))u(x,t) = 0 \text{ on } \mathbb{R}^{n+1} \\
&u(x,t) = \delta(t - x\cdot\omega),\ t << 0
\end{aligned}
\tag{0.1}
$$

where $q(x)$ is a paired conormal distribution with respect to smooth submanifolds $S_2 \subset S_1$. An inverse problem attached to (0.1) is the following: given the scattering kernel $\alpha(s,\theta,\omega)$ for (0.1), what can we determine about the potential $q(x)$? In my thesis, I show that the leading singularities of α determine the singularities of q and moreover that q itself can be recovered up to smoothing terms. The restriction on the orders of the potential, necessary to solve the inverse problem, allow this perturbation to be unbounded. This work develops the mapping properties of specific microlocally-defined operators on various spaces of Lagrangian distributions and Sobolev spaces. Conormal-type distributions that are more singular than q also arise in this thesis and are studied in detail.

Rim Gouia: Inversion of the circular Radon transform on an annulus.

The circular Radon transform Rf puts into correspondence to a given function f its integrals along circular trajectories. The need for such a transform arises in several contemporary problems of medical imaging, synthetic aperture radar and non destructive testing.

The major problems related to the circular Radon transform are the existence and uniqueness of its inversion, inversion formulas and the range description of the transform. In the case, when the circular Radon transform Rf is known for circles of all possible radii, there are well developed theories now addressing most of the questions mentioned above. However, many of these questions are still open when Rf is available for only half of all possible radii, or when the support of f is outside the circle.

The aim of my presentation is to discuss some new results about the existence and uniqueness of the representation of a function by its circular Radon transform with radially partial data. A new inversion formula is presented in the case of the circular acquisition geometry for both interior and exterior problems when the Radon transform is known on an annulus.

Eric Grinberg: : The Admissibility Problem for Radon Transforms on Projective Spaces

The admissibility problem, first posed by I.M. Gelfand in the context of the Plancherel Measure in the context of Lie groups, asks for classification of minimal invertible limited data problems for a given full-data Radon transform. We will discuss the admissibility problem for affine and projective spaces with an emphasis on the projective line transform in a finite projective space of dimension three. This is joint work with David Feldman.

Bin Han: Symmetric Complex Orthonormal Wavelets and Matrix Extension with Symmetry

Abstract: In this talk we shall discuss symmetric orthonormal complex wavelets. It is well-known that except the discontinuous Haar wavelet, compactly supported real-valued orthonormal wavelets cannot have symmetry. In this talk, we first study symmetric orthonormal dyadic complex wavelets such that the orthonormal refinable functions have high linear-phase moments and the wavelets have high vanishing moments. Such wavelets lead to real-valued symmetric tight framelets with desirable moment properties, and are related to coiflets which are of interest in numerical algorithms. Then we shall address symmetric orthonormal complex wavelets with a general dilation factor. This problem is related to the interesting matrix extension problem with symmetry, which plays a fundamental role in many areas. We shall present two families of compactly supported symmetric orthonormal complex wavelets with arbitrarily high vanishing moments or arbitrarily high linear-phase moments.

Alex Iosevich: Regular value theorem in a fractal setting.

The regular value theorem in differential geometry says that if the map F from an n-dimensional smooth manifold X to an m-dimensional smooth manifold Y is a submersion on the set $F^{-1}(y) = \{x \in X : F(x) = y\}$, then $F^{-1}(y)$ is either empty or is a smooth sub-manifold of X of dimension $n - m$. We shall that if Y is replaced by $\mathbb{R}$ and X is replaced by $E \times E'$, where E, E' are compact Ahlfors-David regular subsets of $\mathbb{R}^d$ of Hausdorff dimension s, $0 < s < d$. We shall prove that if F is sufficiently regular and the Hausdorff dimension of E and E' is sufficiently large with respect to the regularity of F, then the upper Minkowski dimension of $\{(x, y) \in E \times E' : F(x, y) = t\}$ is at most $s + s' - 1$. Connections with the theory of regularity of the Fourier integral operators and the Falconer distance conjecture will also be discussed.

Deguang Han: Frames for Group Representations

The Feichtinger frame conjecture states that every bounded (from below) frame is a finite union of basic Riesz sequences. This conjecture turns out to be equivalent to the Kadison- Singer pure state extension problem and several other well-known unsettled problems. In this talk I will focus on a few aspects of this conjecture related to group representation frames and exponential frames for fractal measures. Additionally I will also discuss its connection with a new duality principle for group representations and the II_1 factor classification problem. (This talk is based on collaborations with Dorin Dutkay, Gabriel Picioroaga David Larson, Qiyu Sun and Eric Weber).

Hongyu He: Uniform Bounds on Smooth Matrix Coefficients on L^2-spaces

In this talk, I shall discuss a Cowling-Haagerup-Howe type uniform bound for smooth matrix coefficients of semisimple group acting on L^2-spaces. I will then discuss how the uniform bounds can infer the spectrum of homogeneous spaces.

Ryan Hotovy and Sam Scholze: Unitary Equivalence of Vector Spaces over the Binary Field

Mentored by Dr. David Larson - Texas A&M University

Vector spaces over the binary field $\mathbb{Z}_2$ share certain properties with familiar vector spaces over $\mathbb{R}$ such as the existence of bases for spaces. There are, however, many differences. For example, when equipped with the dot product, a vector space over $\mathbb{Z}_2$ becomes an indefinite inner product space where nonzero vectors may have zero length. We continue previous work on these spaces by investigating subspaces of $\mathbb{Z}_2^n$ and ask when two vector spaces are unitarily equivalent. In particular we consider embeddings of subspaces into $\mathbb{Z}_2^n$ for some n. An algorithm is given showing that every vector space over $\mathbb{Z}_2$ can be embedded in this manner. We also investigate the existence of both Parseval frames and dual frames for vector spaces over Z_2 and their relation to the Grammian operator. Finally we show that, unlike vector spaces over $\mathbb{R}$, the existence of a dual frame pair does not necessarily imply the existence of a Parseval frame of the same length for a space.

John Jasper: Frames with prescribed norms and frame operator

Abstract: We present an extension of Kadisons theorem describing diagonals of self-adjoint operators with 3 point spectrum. As a consequence, we characterize sequence of norms of a frame whose frame operator has 2 point spectrum.

David A. Jimenez: On the Match of Point Configurations

On many computer vision applications, it is often necessary to match a given image to one in an indexed set. One way this may done is by the identification and comparison of landmarks on the image. We call this discrete set of points in $\mathbb{R}^d$ a point configuration. We analyze different algorithms to decide whether or not two point configurations are identical up to a rigid motion. We call this identification, a match. We will also compare and analyze the possibility of using these algorithms for the problem of fingerprint identification.

Victor Kaftal: Equal-norm perturbations of finite frames.

Joint work with David Larson

We construct a map on the collection of n-frames that is

(i) a projection on the collection of equal-norm n-frames;
(ii) every frame and its image under that map share the same frame operator.

In case no two vectors in the frame are orthogonal, we provide an estimate of the distance between the frame and its image.

This map provides a constructive (partial) answer to the so-called Paulsen question of whether a frame that is close to being Parseval and close to being equal norm is close to being a Parseval equal norm frame.

We also generalize this map to projections on a larger class of frames.

Tomoyuki Kakehi: Schroedinger equation on certain compact symmetric spaces and Gauss sum.

Emily King: Generalized Shearlets and the Extended Metaplectic Group

This talk will present new multi-dimensional transformations which generalize shearlet transforms both anisotropically and isotropically to $L^2(\mathbb{R}^d)$. This is achieved by means of explicit constructions of families of reproducing Lie subgroups of the symplectic groups. The properties of these new families will be discussed. In particular, an analog of the Calderón admissibility condition will be provided as well as a discussion of the related co-orbit space theory.

Dominic Kramer: Frame Based Steganography

Steganography is the art of hiding secret data in various data sources such as images. This talk will discuss a technique, which incorporates the use of frames, for hiding an element of a Hilbert space in the coefficients of another element of the space.

Nishu Lal: Product structure of the spectral zeta function of the Sturm-Liouville operator on fractals

In this talk, we will discuss the spectral zeta function of a self-similar Sturm-Liouville operator on the half real line and C. Sabot's work on connecting the spectrum of this operator with the iteration of a rational map of several complex variables. The Sturm-Liouville operator on $[0, \infty)$ is viewed as a limit of the sequence of operators $\frac{d}{dm_{<n>}}\frac{d}{dx}$ with Dirichlet boundary condition on $I_{<n>} = [0, \alpha^{-n}]$ which are the infinitesimal generators of the Dirichlet form $(a_{<n>}, m_{<n>})$. In particular, it is defined in terms of a self-similar measure m and Dirichlet form a, relative to a suitable iterated function system (IFS) on $I = [0, 1]$. In the case of the Sierpinski gasket, as was shown by A. Teplyaev, extending the known relation by M. Lapidus for fractal strings, the spectral zeta function of the Laplacian has a product structure with respect to the iteration of a rational map on the complex plane which arises from the decimation method. In the case of the above self-similar Sturm-Liouville problem, we obtain an analogous product formula, but now expressed in terms of the (suitably defined) zeta function associated with the dynamics of the corresponding 'renormalization map', viewed as a rational function on the 2-dimensional projective space. This is joint work with Michel Lapidus.

David Larson: Operator-Valued Measures, Dilations, and The Theory of Frames

Abstract: We show that there are some natural associations between the theory of frames (including continuous frames and framings), the theory of operator-valued measures on sigma-algebras of sets, and the theory of normal linear mappings between von Neumann algebras. In this connection frame theory itself is identified with the special case in which the domain algebra for the mapping is commutative. Some of the more important results and proofs for mappings in this case extend naturally to the case where the domain algebra is non-commutative. This happens frequently enough, and in profound enough ways, to justify defining a noncommutative frame to be an arbitrary ultraweakly continuous linear mapping between von Neumann algebras. It has been known for a long time that a sufficient condition for a unital bounded linear map between C*-algebras to have a Hilbert space dilation to a bounded homomorphism is that the mapping is that the map be completely bounded. Our theory shows that under suitable hypotheses even if it is not completely bounded it still has a Banach space dilation to a homomorphism, and the Banach space can be rather nice. We view this as a generalization of the known result that arbitrary framings have Banach dilations. This is joint work with Deguang Han, Bei Liu and Rui Liu.

Shidong Li: Fusion frame in action: High resolution image fusion

The notion of Fusion Frames was introduced to study ways in which functions or signals from a set of subspaces can be combined coherently regardless how complicated subspaces are related. We have argued that fusion frames have a deep root in data fusion applications for distributed systems such as sensor networks. It is time that we put fusion frame in action and show exactly how it can be applied in sensor array data fusion. Formulations of high resolution image fusion using fusion frames will be presented. These techniques use the impulse response function of cameras as the building block of the mathematical frames and fusion frames in the fusion process. By taking realistic camera physics into consideration, the proposed approach provides a natural and realistic modeling of the high-resolution image fusion problem. Deterministic and iterative fusion algorithms will be discussed. The fusion frame approach for high-resolution image fusion is also seen to be robust to realistic fusion problems from inhomogeneous image measurements (taken at different space or time or by different cameras). The effectiveness of this approach is demonstrated through both simulated and realistic examples. This is a joint work with Zhenjie Yao and Weidong Yi.

Eileen Martin: Continuously Moving Parseval Frames on Smooth Manifolds.

Moving bases on manifolds are important in the study of differential geometry and are applied in mathematical physics, but moving bases do not exist on all manifolds, for instance, the sphere. An alternative to a moving basis is a Parseval frame of unit-length vectors. We examine the existence of such frames on the Möbius strip, the Klein bottle, and n-dimensional spheres. We prove the existence of a continuously moving, unit-length Parseval frame on S^n when n is an odd integer. More generally, we investigate the relationship between the existence of a nowhere zero vector field and that of a continuously moving Parseval frame of unit length. One potentially useful tool in studying this relationship is the frame force associated with the frame potential. To better understand this possible method, we are led to a study of the dynamical properties of the frame force.

Azita Mayeli: Band-limited wavelets and Besov space norms in Hilbert spaces

In this talk we first introduce inhomogeneous Besov norms for abstract Hilbert spaces. Using band-limited wavelets, we then establish a correlation between the frequency content of a function in a Hilbert space, in terms of its spectral Fourier transform, and its smoothness as described its by Besov norm.

Tadele Mengesha: Local gradient estimates in homogenization of elliptic equations.

For a family of second order linear elliptic equations with rapidly oscillating coefficients, we establish a local uniform $W^{1,p}$ estimates. The coefficients are assumed to be locally sufficiently close to a periodic function. Our approach uses maximal function estimates and Vitali covering lemma from harmonic analysis. We will also use known Lipschitz estimates for elliptic equations with periodically oscillating coefficients.

John Myers: An Algorithm for Unitary Equivalence of Matrices and a Path-Connectedness Application

Two iterative algorithms are developed to transform a given matrix to a unitarily equivalent matrix with constant main diagonal: one if the matrix has elements in $\mathbb{R}$ and one for elements in $\mathbb{C}$. It will be shown that the algorithms will converge in finitely many iterations if the matrices are of size $2^n \times 2^n$. One possible application of the algorithms is in proving that certain sets of projections are path-connected. Hence, continuity is of interest and several results will be given regarding continuity.

Patrick J. Orchard: Orthogonal and Maximal Sets for Bernoulli Measures

Joint work with: Bryan Archer, University of Oklahoma; Rees Dooley, University of Oklahoma; Reid Kelley, University of Oklahoma; Alyssa Leone, University of Oklahoma; Patrick Orchard, University of Oklahoma

We consider orthogonal and maximal sets on $L^2(X_\lambda, \mu_\lambda)$ where μ_λ is the Hutchinson measure associated with the Bernoulli Iterated Function System (IFS) for $\lambda \in (0,1)$ and X_λ is the support of the measure. By a previous theorem from Jorgensen and Pedersen, for $\lambda = \frac{1}{2n}$ we have an orthonormal basis of exponential functions $e^{2\pi i \gamma x}$ where γ is in the preconstructed set $\Gamma_{\frac{1}{2n}}$. We investigate sets $c\Gamma_{\frac{1}{2n}}$ where c is an odd integer dependent on 2n. We prove that the set $3\Gamma_{\frac{1}{4}} \cup \bigcup_{k=1}^{\infty} -4^k(1+3\cdot 4\Gamma_{\frac{1}{4}})$ is an orthogonal and maximal set for the space $L^2(X_{\frac{1}{4}}, \mu_{\frac{1}{4}})$.

Elena Ournycheva: On Y. Nievergelt's inversion formula for the Radon transform

In 1986 Y. Nievergelt suggested a simple formula which allows to reconstruct a continuous compactly supported function on the 2-plane from its Radon transform. This formula falls into the scope of the classical convolution-backprojection method. We show that elementary tools of fractional calculus can be used to obtain more general inversion formulas for the k-plane Radon transform of continuous and L^p functions on R^n for all $1 \le k < n$.

This is a joint work with Boris Rubin.

Nguyen Cong Phuc: Weighted estimates for a nonlinear singular operator and their application

We discuss a global weighted estimate for a class nonlinear singular operators arising from divergence form elliptic operators with BMO coefficients on Reifenberg flat domains. Such an estimate implies new global regularity results in Morrey, Lorentz, and Hölder spaces for solutions of certain quasilinear elliptic equations. Moreover, it can also be used to obtain a capacitary estimate to treat a measure datum quasilinear Riccati type equations with nonstandard growth in the gradient. This talk is based on joint work with Tadele Mengesha.

John Rock: Partition zeta functions of self-similar measures.

For an Iterated Function System (IFS) on the unit interval weighted by a probability vector we define a multifractal spectrum for the self-similar Borel measure uniquely determined by the weighted IFS as the abscissae of convergence of its partition zeta functions. Partition zeta functions are Dirichlet series determined by the self-similar Borel measure and a naturally defined sequence of partitions. These partition zeta functions are indexed by coarse Holder regularity and we show that the corresponding abscissae of convergence equal the Hausdorff dimensions of corresponding Besicovitch subsets of the support of the measure. In the case of the binomial measure, the classical Hausdorff multifractal spectrum is recovered.

José Luis Romero

Coorbit spaces are functional spaces defined by imposing size and integrability conditions to an integral transform T. In the case of the wavelet transform, the corresponding spaces include the family Lebesgue and Sobolev spaces and, more generally, Besov and Tribel-Lizorkin spaces. For the short-time Fourier transform, the associated coorbit spaces are modulation spaces.

Phase-space multipliers are operators defined by applying a mask to the transform T. I will present a result on the characterization of membership to a certain coorbit space in terms of families of phase-space multipliers. In the case of time-frequency analysis, this generalizes recent results to an irregular context.

Lakshmi Roychowdhury: Optimal points for any Cantor distribution

For over fifty years electrical engineers and mathematicians have been interested in the problem of efficiently "quantizing" a probability distribution in the sense of estimating a given probability by a discrete probability supported on a finite set. This problem arises in signal processing, data compression, cluster analysis, and pattern recognition, and it also has been studied in the context of economics, statistics, and numerical integration. Given a Borel probability measure P on $\mathbb{R}^d$, the nth *quantization error* is defined by

$$V_n = \inf_{\alpha} \int \min_{a \in \alpha} \|x - a\|^2 dP(x),$$

where the infimum is taken over all subsets α of $\mathbb{R}^d$ with card $\alpha \leq n$ for $n \geq 1$. We note that if $\int \|x\|^2 dP(x) < \infty$ then there is some set α for which the infimum is achieved. The set α for which the infimum is achieved is called the optimal set of n-means or n-optimal set. It can be shown that for a continuous probability measure P an optimal set of n-means always has exactly n-elements. Let $S_1, S_2 : \mathbb{R} \to \mathbb{R}$ be defined by

$$S_1(x) = \frac{1}{3}x \text{ and } S_2(x) = \frac{1}{3}x + \frac{2}{3}.$$

Then for a probability vector (p_1, p_2) with $p_1, p_2 > 0$, there exists a unique Borel probability measure P on $\mathbb{R}$ such that $P = p_1 P \circ S_1^{-1} + p_2 P \circ S_2^{-1}$, where $P \circ S_i^{-1}$ denotes the image measure of P with respect to S_i for $i = 1, 2$. For $\sigma = (\sigma_1, \sigma_2, \cdots, \sigma_k) \in \{1, 2\}^k$, set $S_\sigma = S_{\sigma_1} \circ \cdots \circ S_{\sigma_k}$ and $J_\sigma = S_\sigma([0, 1])$. Then the set $C = \prod_{k \in \mathbb{N}} \bigcup_{\sigma \in \{1,2\}^k} J_\sigma$ is the Cantor set and equals the support of the probability measure P. In this talk, I will show what are the n-optimal sets, and the corresponding nth quantization error for $n \geq 1$.

Mrinal Kanti Roychowdhury: Quantization dimension for infinite self-similar mappings

The term 'quantization' in the title originates in the theory of signal processing and denotes a process of discretising signals. As a mathematical theory quantization concerns the best approximation of probabilities by discrete probabilities with a given number of points in their support. Given a Borel probability measure μ on $\mathbb{R}^d$, a number $r \in (0, +\infty)$ and a natural number $n \in \mathbb{N}$, the nth *quantization error* of order r for μ is defined by

$$V_{n,r}(\mu) = \inf\{\int d(x, \alpha)^r d\mu(x) : \alpha \subset \mathbb{R}^d, \text{card}(\alpha) \leq n\},$$

where $d(x, \alpha)$ denotes the distance from the point x to the set α with respect to a given norm $\|\cdot\|$ on $\mathbb{R}^d$. Note that if $\int \|x\|^r d\mu(x) < \infty$ then there is some set α for which the infimum is achieved. The *quantization dimension of order* r for μ is defined to be

$$D_r = D_r(\mu) = \lim_{n\to\infty} \frac{r \log n}{-\log V_{n,r}(\mu)},$$

if the limit exists. One sees that the quantization dimension is actually a function $r \mapsto D_r$ which measures the asymptotic rate at which $e_{n,r}$ goes to zero. Determination of the quantization dimension for a probability measure generated by an infinite iterated function system associated with a probability vector is a long-time open problem. Recently, I have determined the quantization dimension for a probability measure generated by an infinite contractive similarity mappings $S_1, S_2, \cdots$ associated with a probability vector $(p_1, p_2, \cdots)$, that is, the probability measure μ satisfies $\mu = \sum_{j=1}^{\infty} p_j \mu \circ S_j^{-1}$. In this talk, I will discuss about it.

Boris Rubin: The Funk, Cosine and Sine transforms on Stiefel and Grassmann manifolds.

The Funk transform takes a function f on the unit sphere in R^3 to a function on the set of great circles as an integral of f over the corresponding great circle. This transform can be regarded as a member of the analytic family of normalized cosine transforms. We extend basic facts about the Funk transform and related analytic families of cosine and sine transforms to the more general context for Stiefel or Grassmann manifolds.

Dmitry Ryabogin: A counterexample to a problem of Klee

A question of Klee asks whether a convex body is uniquely determined (up to a translation and reflection in the origin) by its inner section function. We construct a counterexample showing that this is not true. This is a joint work with R. J. Gardner, V. Yaskin, A. Zvavitch.

Myung-Sin Song: Matrix Factorization and Lifting I

Joint work with P. E. T. Jorgensen.

As a result of recent interdisciplinary work in signal processing (audio, still-images, etc), a number of powerful matrix operations have led to advances both in engineering applications and in mathematics. Much of it is motivated by ideas from wavelet algorithms. The applications are convincing measured against other processing tools already available, for example better compression. We develop a versatile theory of factorization for matrix functions. By a matrix valued function we mean a function of one or more complex variables taking values in the group GL_N of invertible $N \times N$ matrices. Starting with this generality, there is a variety of special cases, also of interest, for example, one variable, or restriction to the case $n = 2$; or consideration of subgroups of GL_N or SL_N, i.e., specializing to the case of determinant equal to one. A number of factorization theorems and sketch their applications to signal (image processing) in the framework of multiple frequency bands will be presented.

Jeehyeon Seo: Bi-Lipschitz embeddability of the Grushin plane into Euclidean space.

Many sub-Riemannian manifolds like the Heisenberg group do not admit bi-Lipschitz embedding into any Euclidean space. In contrast, the Grushin plane admits a bi-Lipschitz embedding into some Euclidean space. This is done by extending a bi-Lipschitz embedding of the singular line, using a Whitney decomposition of its complement.

Plamen Stefanov: Tensor Tomography

We will present a survey of the recent results, obtained jointly with Gunther Uhlmann, on the problem of recovery a tensor field on a Riemannian manifold with boundary from integrals along geodesics. This problem appears also as a linearization of the boundary and the lens rigidity problems - recovery of a metric from travel times or the scattering relation on the boundary. We use mostly microlocal methods.

Alex Stokolos: Bellman Function for the Dyadic Maximal Operator

Bellman Function for the $L^2 \to L^1$ Dyadic Maximal Operator inequality is found by A. Melas in a brilliant paper [1]. The proof is long and involved. We propose a short and very natural proof of Melas' result based on the method invented by L.Slavin, A.Stokolos and V.Vasyunin [2]. The resulting function came out as a solution to a boundary value problem for the Monge-Ampère PDE. This is a joint work with

Shijun Zheng and V.Maymeskul.

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Tara D. Taylor: Using Cantor Sets to Study the Connectivity of Sierpiński Relatives.

This paper presents an exploration of the connectivity of the class of fractals known as the Sierpiński relatives. The Sierpiński gasket (or triangle) is the most well-known relative. The relatives are attractors of iterated function systems that involve the same contractive mappings as for the gasket, combined with symmetries of the square. These relatives all have the same fractal dimension, but different topologies. Some are completely disconnected, some are simply-connected, and some are multiply-connected. For some of the relatives, one can determine the connectivity by considering certain Cantor sets that are subsets. These Cantor sets are variations of the usual middle thirds Cantor set, and can be viewed in binary or quaternary instead of ternary.

Yang Wang: Peano curves for fractals

Shijun Zheng: Semilinear Schroedinger Equation with Magnetic Potentials

In quantum physics the magnetic Schroedinger equation provides an ideal model when a Bose-Einstein Condensate is confined or released. As we will understand, the attractive or repulsive property of the potential has implications in geometry as well as in physics.

We will present the analytic approach to show the existence of the solution within its lifespan. The techniques from Harmonic Analysis include Strichartz and smoothing estimates, Besov and Sobolev embeddings for a magnetic perturbation, and, bilinear estimates for fractional derivatives. Time permitting we will also briefly review recent development for semilinear Schroedinger and wave equations on Riemannian symmetric spaces.