On the distance between homotopy classes of maps taking values in manifolds

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Abstract

It is well known that for $p \geq m$ the degree of maps in $W^{1,p}(S^m, S^m)$ is well defined and one has the following decomposition of this space as a disjoint union of homotopy classes: $W^{1,p}(S^m, S^m) = \bigcup_{d \in \mathbb{Z}} \mathcal{E}_d$. It is natural then to study the distance $\delta_p(d_1, d_2)$ between each pair of distinct homotopy classes $\mathcal{E}_{d_1}$ and $\mathcal{E}_{d_2}$, defined by

$$\delta_p(d_1, d_2) = \inf \left\{ \int_{S^m} |\nabla (u_1 - u_2)|^p : u_1 \in \mathcal{E}_{d_1}, u_2 \in \mathcal{E}_{d_2} \right\}.$$

In the one dimensional case ($m = 1$) we find that the distance is given explicitly by the formula $\delta_p(d_1, d_2) = \frac{2^{1+1/p}|d_2 - d_1|}{\pi^{1-1/p}}$.

In higher dimensions, $m \geq 2$, it turns out that in the limiting case $p = m$, the distance between the homotopy classes is always zero. On the other hand, when $p > m$, for $d_1 \neq d_2$ the distance is positive, but independent of $d_1$ and $d_2$, i.e., $\delta_p(d_1, d_2) = c(m, p)$. Here $c(m, p)$ is a positive constant that had already been computed explicitly by Talenti (for $m = 2$) and Cianchi (for any $m$) in the context of Sobolev-type inequalities on spheres.

This talk is based on a work in progress with Shay Levy and on an earlier work with Jacob Rubinstein.

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