1. State from memory how you can evaluate a line integral.
2. Explain how a line integral generalizes a definite integral known from calculus.
3. What do we mean by path independence of a line integral? Why is this practically important?
   What does it mean physically?
4. List the kinds of integral we have discussed in this chapter.
5. How can we convert line integrals to surface integrals? State the conditions in the corresponding theorem.
6. How can we convert surface integrals to volume integrals?
7. What role did the gradient play in this chapter?
8. How is the divergence defined and how did we use it in this chapter?
9. Why was it essential in Stokes’s theorem whether we used right-handed or left-handed coordinates?
10. What are typical applications of line integrals? Of surface integrals?
11. Orientation played a role in connection with surface integrals. Explain.
12. What is a smooth surface? A piecewise smooth surface? Give examples. Why did we need these concepts?
13. State Laplace’s equation. Where in physics is it important?
14. Summarize our discussions on harmonic functions.
15. In line and surface integrals we used vector functions \( \mathbf{F} \), but the integrands were actually scalar functions. Why did we proceed in this way (which at first sight looks somewhat like a detour)?

**Line Integrals**

\[ \int_C \mathbf{F} \cdot \mathbf{dr} \]. *Work Done by a Force.* In Problem 16-30, with \( \mathbf{F} \) and \( C \) as given, evaluate this integral by the method that seems most suitable (direct integration, use of exactness or Green’s theorem or Stokes’s theorem). Recall that if \( \mathbf{F} \) is a force, the integral gives the work done in a displacement. (Show the details of your work.)

16. \( \mathbf{F} = [x^2, -2y^2] \), \( C \) the straight-line segment from \((4, 2)\) to \((-3, 5)\)
17. \( \mathbf{F} = [xy, y x, x y^2] \), \( C \) the straight-line segment from \((\pi, 1, 0)\) to \((\frac{1}{2}, \pi, 1)\)
18. \( \mathbf{F} = [xy, z, 0] \), \( C: y = 2x^2, z = x \) from \((1, 2, 1)\) to \((2, 8, 2)\)
19. \( \mathbf{F} = [y^2, 2xy + \sin x, 0] \), \( C \) the boundary of \(0 \leq x \leq \pi/2, 0 \leq y \leq 2, z = 0\)
20. \( \mathbf{F} = [-y^2, x^2 + e^x, 0] \), \( C \) the circle \(y^2 + z^2 = 4, z = 0\)
21. \( \mathbf{F} = [\cos \pi y, \sin \pi x, \cos \pi x] \), \( C \) the boundary of \(0 \leq x \leq 1/2, 0 \leq y \leq 4, z = x\)
22. \( \mathbf{F} = [-z, 5x, -y] \), \( C \) the ellipse \(x^2 + y^2 = 4, z = x + 2\)
23. \( \mathbf{F} = [8xy, 4x^2, 2 \cos 2z] \), \( C \) the helix \( r = (\cos t, \sin t, t), 0 \leq t \leq \pi/4\)
24. \( \mathbf{F} = [xe^{zy}, 2 \sin 2y, xe^{xy}] \), \( C \) the parabola \(y = x, z = x^2, -1 \leq x \leq 1\)
25. \( \mathbf{F} = [e^x - e^y, e^y, e^x] \), \( C: x = \ln y, y = \ln x, 1 \leq x \leq 2\)
26. \( \mathbf{F} = [x \ln y, 2ye^x, 0] \), \( C \) the boundary of the rectangle \(0 \leq x \leq 2, 1 \leq y \leq 2, z = 0\)
27. \( \mathbf{F} = [x^3, 3xe^y, 0] \), \( C: x^2 + y^2 = 9, z = x^2\)
28. \( \mathbf{F} = [\cos y, x \sin z, z] \), \( C: r = (t, 3t, t^2), 0 \leq t \leq 1\)
29. \( \mathbf{F} = [\sin \pi x, z, 0] \), \( C \) the boundary of the triangle with vertices \((0, 0, 0), (1, 0, 0), (1, 1, 0)\)
30. \( \mathbf{F} = [x^2, y^2, y^2 z] \), \( C \) the helix \( r = (\cos t, \sin t, 3t), 0 \leq t \leq \pi/2\)

**Double Integrals. Center of Gravity.** Find the coordinates \( \bar{x}, \bar{y} \) of the center of gravity of a mass of density \( f(x, y) \) in the region \( R \). (This amounts to the evaluation of double integrals. Show the details. Sketch \( R \).)

31. \( f = xy, \quad R: 0 \leq x \leq 1, 0 \leq y \leq x \)
32. \( f = x^2y, \quad R: -1 \leq x \leq 1, x^2 \leq y \leq 1 \)
33. \( f = 1, \quad R: 1 \leq x \leq 2, 0 \leq y \leq \ln x \)
34. \( f = 1, \quad R: x^2 + y^2 \leq a^2, y \geq 0 \)
35. \( f = x^2, \quad R: -1 \leq x \leq 2, x^2 \leq y \leq x + 2 \)
36. \( f = x^2 + y^2, \quad R: x^2 + y^2 \leq a^2, x \geq 0, y \geq 0 \)

**Surface Integrals**

\( \iint_S \mathbf{F} \cdot \mathbf{n} \, dA \). *Divergence Theorem.* Evaluate this integral directly or, if possible, by the divergence theorem. (Show the details.)

37. \( \mathbf{F} = [x, y], \quad S: z = 2x + 5y, 0 \leq x \leq 2, -1 \leq y \leq 1 \)
38. \( \mathbf{F} = [\sin x, z, y], \quad S: y^2 + z^2 = 4, -1/2 \leq x \leq 1/2, y \geq 0, z \geq 0 \)
39. \( \mathbf{F} = [0, 20y, 2z^3], \quad S \) the surface of \( 0 \leq x \leq 6, 0 \leq y \leq 1, 0 \leq z \leq y \)
40. \( \mathbf{F} = [x^3, y^3, z^4], \quad S \) the sphere \( x^2 + y^2 + z^2 = 4 \)
41. \( \mathbf{F} = [0, x^2, -xyz], \quad S: \mathbf{r} = (u, u^2, v), 0 \leq u \leq 1, -2 \leq v \leq 2 \)
42. \( \mathbf{F} = [e^y, 0, ze^x], \quad S: \mathbf{r} = (u, 2u, v), -1 \leq u \leq 1, 0 \leq v \leq 3 \)
43. \( \mathbf{F} = [1, 1, 1], \quad S: x^2 + y^2 + 4z^2 = 4, z \geq 0 \)
44. \( \mathbf{F} = [y^2, x^2, z^4], \quad S \) the surface of the cylinder \( x^2 + y^2 \leq 4, 0 \leq z \leq 2 \)
45. \( \mathbf{F} = [x + z, y + z, x + y], \quad S \) the sphere \( x^2 + y^2 + z^2 = 9 \)