10. PROJECT. Even and Odd Functions. (a) Are the following expressions even or odd?
Sums and products of even functions and of odd functions. Products of even times odd functions.  
Absolute values of odd functions. \( f(x) + f(-x) \) and \( f(x) - f(-x) \) for arbitrary \( f(x) \).
(b) Write \( e^{kx} \), \( 1/(1 - x) \), \( \sin (x + k) \), \( \cosh (x + k) \) as sums of an even and an odd function.
(c) Find all functions that are both even and odd.
(d) Is \( \cos^2 x \) even or odd? \( \sin^2 x \)? Find the Fourier series of these two functions. Do you recognize familiar identities?

Fourier Series of Even and Odd Functions
State whether the given function is even or odd. Find its Fourier series. Sketch the function and some partial sums. (Show the details of your work.)

11. \( f(x) = \begin{cases} k & \text{if } -\pi/2 < x < \pi/2 \\ 0 & \text{if } \pi/2 < x < 3\pi/2 \end{cases} \)
12. \( f(x) = \begin{cases} -2x & \text{if } -\pi < x < 0 \\ 2x & \text{if } 0 < x < \pi \end{cases} \)
13. \( f(x) = \begin{cases} x & \text{if } -\pi/2 < x < \pi/2 \\ \pi - x & \text{if } \pi/2 < x < 3\pi/2 \end{cases} \)
14. \( f(x) = \begin{cases} x & \text{if } 0 < x < \pi \\ \pi - x & \text{if } \pi < x < 2\pi \end{cases} \)
15. \( f(x) = x^2/2 \) \((-\pi < x < \pi)\)
16. \( f(x) = 3x(\pi^2 - x^2) \) \((-\pi < x < \pi)\)

Show that

17. \( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4} \) (Use Prob. 11.)
18. \( 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots = \frac{\pi^2}{6} \) (Use Prob. 15.)
19. \( 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots = \frac{\pi^2}{12} \) (Use Prob. 15.)

Half-Range Expansions
Find the Fourier cosine series as well as the Fourier sine series. Sketch \( f(x) \) and its two periodic extensions. (Show the details.)
20. \( f(x) = 1 \) \((0 < x < L)\)
21. \( f(x) = x \) \((0 < x < L)\)
22. \( f(x) = x^2 \) \((0 < x < L)\)
23. \( f(x) = \pi - x \) \((0 < x < \pi)\)
24. \( f(x) = x^3 \) \((0 < x < L)\)
25. \( f(x) = e^x \) \((0 < x < L)\)

10.5 Complex Fourier Series. Optional
In this optional section we show that the Fourier series

\[ f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \]

can be written in complex form, which sometimes simplifies calculations (see Example 1, below). This is done by the Euler formula (5), Sec. 2.3, with \( nx \) instead of \( x \), that is,

\[ e^{inx} = \cos nx + i \sin nx, \]

\[ e^{-inx} = \cos nx - i \sin nx. \]