**Problem Set 9.7**

**Application of Triple Integrals**

**Mass Distribution.** In Pros. 1–7 find the total mass of a mass distribution of density \( \sigma \) in a region \( T \) in space. (Show the details of your work.)

1. \( \sigma = x^2 + y^2 + z^2 \), \( T \) the box \( |x| \leq 1 \), \( |y| \leq 3 \), \( |z| \leq 2 \)
2. \( \sigma = e^{-x-y-z} \), \( T: 0 \leq x \leq 1 - y \), \( 0 \leq y \leq 1 \), \( 0 \leq z \leq 2 \)
3. \( \sigma = \sin \pi x \cos \pi y + 2 \), \( T \) the box \( 0 \leq x \leq 1 \), \( 0 \leq y \leq \frac{1}{2} \), \( |z| \leq 2 \)
4. \( \sigma = \frac{1}{2}(x^2 + y^2)^2 \), \( T \) the cylinder \( x^2 + y^2 \leq 9 \), \( -3 \leq z \leq 3 \)
5. \( \sigma = 12xy \), \( T \) the tetrahedron with vertices \((0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)\)
6. \( \sigma = 4z \), \( T \) the region in the first octant bounded by \( y = 1 - x^2 \) and \( z = x \)
7. \( \sigma = 2yz \), \( T: 0 \leq x \leq 2 \), \( e^{-x} \leq y \leq 1 \), \( e^{-x} \leq z \leq 1 \)

**Moment of Inertia.** Find the moment of inertia \( I_z = \int \int \int_T (y^2 + z^2) \, dx \, dy \, dz \) of a mass of density 1 in \( T \) about the \( x \)-axis, where \( T \) is

8. The box \( 0 \leq x \leq a \), \( -b/2 \leq y \leq b/2 \), \( -c/2 \leq z \leq c/2 \)
9. The cube \( 0 \leq x \leq a \), \( 0 \leq y \leq a \), \( 0 \leq z \leq a \)
10. The cone \( y^2 + z^2 \leq x^2 \), \( 0 \leq x \leq h \)
11. The cylinder \( y^2 + z^2 \leq a^2 \), \( 0 \leq x \leq h \)
12. The ball \( x^2 + y^2 + z^2 \leq a^2 \)

**Application of the Divergence Theorem**

**Surface Integral** \( \iint_S \mathbf{F} \cdot \mathbf{n} \, dA \). Evaluate this integral by the divergence theorem for the following data. (Show the details. More such problems in the next problem set.)

13. \( \mathbf{F} = [x^2, 0, z^2] \), \( S \) the surface of the box in Prob. 1
14. \( \mathbf{F} = [x^2, e^y, 0] \), \( S \) the surface of the cube \( |x| \leq 1 \), \( |y| \leq 1 \), \( |z| \leq 1 \)
15. \( \mathbf{F} = [\cos y, \sin x, \cos z] \), \( S \) the surface of \( x^2 + y^2 \leq 4 \), \( |z| \leq 2 \)
16. \( \mathbf{F} \) as in Prob. 15, \( S \) the surface of \( x^2 + y^2 \leq 9 \), \( 0 \leq z \leq 2 \)
17. \( \mathbf{F} = [4x, x^2y, -x^2z] \), \( S \) the surface of the tetrahedron in Prob. 5
18. \( \mathbf{F} = [2x^2, \frac{3}{2}y^2, -\cos \pi z] \), \( S \) as in Prob. 5
19. \( \mathbf{F} = [x^3, y^3, z^3] \), \( S \) the sphere \( x^2 + y^2 + z^2 = 9 \)

20. **WRITING PROJECT.** **Simplifications** occur in the use of the divergence theorem if \( \text{div} \, \mathbf{F} \) is constant. Find other cases of simplifications (\( \text{div} \, \mathbf{F} \) depending on one variable only, symmetry, the use of cylindrical or spherical coordinates—in Prob. 19, e.g.—etc.). Invent examples. Write down your results systematically in a short essay.