MATH 1552-02 Answers to Sample Test 1

June 16, 2004

1. Use integration by parts to get
$$\int x\cos\pi x\,dx = \frac{\cos\pi x}{\pi^2} + \frac{x\sin\pi x}{\pi} + C.$$

2. Use substitution $u = x^2$ to get
$$\int x^5 e^{x^2}\,dx = \frac{1}{2}\int u^2 e^u\,du,$$
and then integrate this by parts twice to get
$$\frac{1}{2}e^{x^2}(2 - 2x^2 + x^4) + C.$$

3. Use double-angle formulas to get
$$\int_0^{\pi/2} \sin^2 x\cos^2 x\,dx = \frac{\pi}{16}.$$

4. Save $\sec^2 t$ and express $\sec^4 t$ in terms of $\tan t$, and then use $u = \tan t$ to get
$$\tan t + \frac{2}{3}\tan^3 t + \frac{1}{5}\tan^5 t + C.$$

5. Use substitution $x = \frac{3}{4}\sec u$ to obtain
$$\int \frac{dx}{x^2\sqrt{16x^2-9}} = \frac{4}{9}\sin u + C.$$
Then use a right triangle to express this in terms of x as
$$\frac{\sqrt{16x^2-9}}{9x} + C.$$

6. Factor the denominator and use partial fractions to get
$$\int \frac{x-1}{x^3+x}\,dx = \arctan x - \ln x + \frac{1}{2}\ln(1+x^2) + C.$$

7. Use long division and factor the denominator. Then use partial fractions to get
$$\frac{x^3}{x^2+4x+3} = x - 4 - \frac{1}{2(1+x)} + \frac{27}{2(3+x)}.$$

8. Integrate by parts to get
$$\int_1^\infty \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty}\left(-\frac{1}{x} - \frac{\ln x}{x}\right)\Big|_1^t = 1.$$

9. The graph of the equation
$$4x^2 + 4y^2 + 4z^2 - 8x + 16y = 1$$
is the sphere centered at $(1, -2, 0)$ of radius $\frac{\sqrt{21}}{2}$.

10. The horizontal component is $50\cos 38^\circ \approx 39.4$ N and the vertical component is $50\sin 38^\circ \approx 30.8$ N.