MATH 1552-02 Answers to Sample Test 2

June 30, 2004

1. Represent the diagonal of the cube as $\langle 1,1,1\rangle$ and one of its edges by $\langle 1,0,0\rangle\rangle$ and then use the dot product to compute the the cosine of the angle to be $\sqrt{3}/3$. Conclude that the angle beween the two vectors is $\arccos\frac{\sqrt{3}}{3}$, which is approximately 58 degrees.

2. Compute the cross-product of the vectors $\vec{PQ}$ and $\vec{PR}$ and divide its length by two to get $\sqrt{6}/2$.

3. Find two points on the line; for example $(0,0,0)$ and $(6,3,2)$. Use these two points together with $(1,-1,1)$ to find two vectors in the plane, and compute their cross-product to get a vector normal to the plane. Use the normal vector and any of the three points above to get the equation of the plane: $5x-4y-9z=0$.

4. Find a point on one plane, say, $(0,0,-1)$, and compute its distance to the other plane using the point-plane distance formula. That requires a point on the other plane, say, $(0,0,-4/9)$ and a normal vector to either plane, say, $\langle 1,2,-3\rangle$. The distance between the planes comes to
$$\frac{|\langle 0,0,-1+4/9\rangle\cdot\langle 1,2,-3\rangle|}{|\langle 1,2,-3\rangle|}=\frac{5}{3\sqrt{14}}.$$

5. The distance from $P(x,y,z)$ to the x-axis is $\sqrt{y^2+z^2}$, while the distance from P to the yz-plane is $|x|$. Setting these equal to each other and squaring both sides of the equation, we get $y^2+z^2=x^2$. That is an equation of the cone whose axis of symmetry is the x-axis and whose surface forms the angle of $\pi/4$ with the x-axis.

6. Since $x=\ln t$, we have $t=e^x$, and so $y=\sqrt{e^x}=e^{x/2}$. Also, $t\geq 1$ implies that $x\geq 0$. Hence the graph is the familiar exponential curve starting at $(0,1)$ a proceeding right. It would take me too much time to figure out how to embed the graphic image in this pdf file, sorry.

7. Find the second derivative of y with the respect to x to be
$$-\frac{2(4+t^2)}{9(t^2-4)^3}.$$
This expression is positive whenever the denominator is negative (the numerator is always positive and there is a minus sign in front of the entire expression). The denominator is negative, and the function is concave upward, when $-2<t<2$.

8. Use the formula for the length of the curve in parametric form to get the length $4\sqrt{2}-2$.

9. Find dy/dx to be
$$\frac{\cos 2t+\sin 2t}{\cos 2t-\sin 2t}.$$
This expression is zero when $t=3\pi/8$ or $t=7\pi/8$, and is undefined when $t=\pi/8$ or $t=5\pi/8$. The first pair of values of t corresponds to points $(0.5,1.20711)$ and $(0.5,-0.20711)$, while the second pair corresponds to points $(1.20711,0.5)$ and $(-0.20711,0.5)$.

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10. The polar equation of the circle of radius a is $r = a$. Integrate the expression for the length of the polar curve over $0 \leq \theta \leq 2\pi$ to get $2\pi a$.