6th Southeastern Lie Theory Workshop
Noncommutative Geometry and Representation Theory

May 10–12, 2013 • Louisiana State University • Baton Rouge

Invited talks

	Friday, May 10 *Nicholson 109*	*Saturday, May 11* *Design 103*	*Sunday, May 12* *Lockett 5*
9:00–9:50	Vogan	Knutson	Zhang
10:00–10:50	Anno	Rogalski	Zhu
	Coffee		
11:20–12:10	Geiß	Speh	Schedler
	Lunch		
2:00–2:50	Stanton	Przebinda	
2:50–3:20	*Coffee*		

Friday afternoon: AIM-style sessions

	Lockett 233	*Nicholson 109*
3:20–4:35	Sierra	Braden/ Mautner
	Break	
4:45–6:00	Sierra	Braden/ Mautner

Saturday afternoon: Contributed talks

	Lockett 233	*Lockett 282*	*Lockett 321*
3:30–6:00 (25-min. talks)	Vega Talian Brown Gunningham Rider	Harris Bremer Brice Li Thompson	Rupel Johnson Stella Conner Muller
7:00	*Conference Dinner* Bay Leaf Indian Restaurant 5160 S. Sherwood Forest Blvd., Baton Rouge		

Abstracts for invited talks

Rina Anno (Pittsburgh) **DG approach to Fourier-Mukai transforms**

Working with DG enhancements of triangulated categories resolves many difficulties specific for triangulated categories. Thus, when studying derived categories of sheaves on schemes and Fourier-Mukai transforms between them, we can replace the categories by suitable DG categories of modules, and the kernels of FM functors by DG-bimodules. Thus all constructs with FM transforms that involve adjunction morphisms behave in some canonical way, since there are canonical diagrams of bimodules behind them.

Christof Geiß (UNAM) **The representation type of non-degenerate QPs**

This is a report on joint work with Daniel Labardini-Fragoso and Jan Schröer.

Non-degenerate Quivers with Potential (QP) play an important role in the study of cluster algebras with skew symmetric exchange matrix. See for example the two fundamental papers by Derkesn-Weyman-Zelevinsky on this subject.

Since the Jacobian algebra of a QP is the inverse limit of finite-dimensional algebras Drozd's famous tame-wild dichotomy applies here. We will recall the precise definitions.

Theorem. Let (Q, S) be a non-degenerate connected QP over an algebraically closed field, and A its Jacobian algebra. Then the following holds:

a) A is wild if Q is not mutation finite or if Q is mutation equivalent to one of the quivers X_6, X_7 or a m-Kronecker with m at least 3.

b) If Q comes from a triangulation of the torus with one marked point (with or without boundary) then A can be wild or tame.

c) If Q is none of the quivers described in a) and b) then A is tame.

If times permits, we will discuss also the uniqueness of non-degenerate potentials for most of the mutation finite quivers.

Allen Knutson (Cornell) **Bruhat atlases on flag manifolds and wonderful compactifications**

Bruhat cells are finite-dimensional vector spaces with beautiful stratifications, even when they live in non-affine Kac-Moody flag manifolds. Can we study other famous stratified manifolds using them?

Define a Bruhat atlas on M to be an atlas of open sets on M, each carrying a stratified isomorphism to a Bruhat cell for some Kac-Moody group G. I'll explain how to look for such a group, and describe a construction of Bruhat atlases for partial flag manifolds H/P (with Lusztig's projected Richardson stratification), and wonderful compactifications (with the double Bruhat stratification).

This is joint work with Xuhua He and Jiang-Hua Lu, and (for the H/B case) with Alex Woo and Alex Yong.

Tomasz Przebinda (Oklahoma) **Howe correspondence and Springer correspondence**

This is a joint work with Anne-Marie Aubert and Witold Kraskiewicz.

Dan Rogalski (UC San Diego) **Classifying orders in the Sklyanin algebra**

This is joint work with Sue Sierra and Toby Stafford. One of the major open problems in noncommutative algebraic geometry is the classification of noncommutative surfaces, and this work resolves a significant case of the problem. Namely, let S be the 3-dimensional Sklyanin algebra, and let T be its 3-veronese ring. Then we show how to describe all maximal orders which are contained in T, as certain kinds of blowups of T. The same techniques also apply to other noncommutative surfaces containing a smooth elliptic curve as a divisor.

Travis Schedler (UT Austin) **The Harish-Chandra $\mathcal{D}$-module and generalizations**

The Harish-Chandra $\mathcal{D}$-module on a semisimple Lie algebra $\mathfrak{g}$ is a fundamental, well-studied object in representation theory: the Springer correspondence relates its simple summands to representations of the Weyl group; it is also closely related to character sheaves. Its definition by Hotta and Kashiwara is expressed via radial parts of G-invariant differential operators on $\mathfrak{g}$. In this talk I will recall this and relate it to the Poisson homology of Slodowy slices of coadjoint orbits in $\mathfrak{g}$ on the one hand, and the de Rham cohomology of their Springer resolutions on the other hand. I will explain a conjectural generalization to the parabolic Springer resolution, where one studies the Harish-Chandra $\mathcal{D}$-module on nilpotent orbit closures or their universal Poisson deformations. This includes joint work with Etingof and with Bellamy.

Birgit Speh (Cornell) **Branching laws for infinite dimensional representations**

I will discuss the restriction of unitary and nonunitary representations of a group semisimple G to a subgroup H. My main example discussed will be the restriction of the principal series representations of the orthogonal group $O(n,1)$ to the subgroup $O(n-1,1)$. This is joint work with T. Kobayashi.

Robert Stanton (Ohio State) **Geometry of special symplectic representations**

We will describe a class of symplectic Lie algebra representations over any field of characteristic not 2 or 3 that have many of the exceptional algebraic and geometric properties of symmetric three forms in two dimensions. The main algebraic result is how suitably generic elements of these representation spaces can be uniquely written as the sum of two elements of a naturally defined Lagrangian subvariety. This leads to the existence of special geometric structure on appropriate subsets of the generic elements. Over the real numbers this structure reduces to either a conic, special pseudo-Kähler metric or a conic, special para-Käbler metric.

David Vogan (MIT) **Extended reductive groups and representation theory**

Suppose G is a complex connected reductive algebraic group, and Σ is a finite group. An "extended group" is an extension of Σ by G; that is, a group having G as a normal subgroup, with quotient Σ. One of the most important examples of such extended groups is Langlands' L-group, which is an extension of a Galois group by a reductive group.

I will discuss several ways in which extended groups arise in parametrizing rational forms and representations of reductive groups. Then I'll discuss some problems arising in recent joint work with Lusztig about the representation theory of extended groups themselves.

James Zhang (Washington) **Discriminant controls automorphism group of noncommutative algebras**

We use discriminant to compute the automorphism group of noncommutative algebras.

Xinwen Zhu (Northwestern) **Non-abelian Hodge theory for curves in characteristic p**

Let G be a reductive group and X be a smooth projective curve over an algebraically closed field of characteristic $p > 0$. I will give a description of the moduli space of de Rham G-local systems on X in terms of the moduli space of G-Higgs fields on X. This description can be regarded as a positive characteristic non-abelian Hodge theory for curves.

Abstracts for AIM-style sessions

Sue Sierra (Edinburgh) **Noncommutative algebraic geometry: moduli theory and applications**

A fundamental technique of noncommutative algebraic geometry is to study an $\mathbb{N}$-graded noncommutative algebra R through the moduli spaces parameterising graded R-modules, particularly so-called point modules. Relating algebraic geometry to ring theory has led to many advances in noncommutative algebra. In these lectures we'll present the techniques of moduli theory in noncommutative algebra and give several important applications. These include Artin, Tate, and Van den Bergh's proof that 3-dimensional Sklyanin algebras are noetherian (the origin of the well-known slogan that "a noncommutative $\mathbb{P}^2$ contains an elliptic curve") and a recent result with Walton that the enveloping algebra of the Virasoro algebra is not noetherian. This last result settled a 23-year old question in Lie theory.

Tom Braden (UMass Amherst) & Carl Mautner (Harvard) **Working with parity sheaves**

The recent theory of parity sheaves allows one to generalize arguments in geometric representation theory to positive characteristic. We will explore the basic features of this theory, study useful computational techniques and discuss connections to the representation theory of reductive groups.

Abstracts for contributed talks

Chris Bremer (LSU) **A theory of minimal K-types for flat G-bundles**

Let G be a reductive algebraic group defined over $\mathbb{C}$. Recent work on the geometric Langlands conjecture has shown that there is a relationship between local systems with coefficients in G (i.e., flat G-bundles) and representations of affine Kac-Moody algebras. Much less is known about flat G-bundles with singularities outside of the tamely ramified case, especially those with irregular singular points.

This talk will illuminate the local structure of highly singular flat G-bundles by drawing an analogy with the work of Moy and Prasad on cuspidal representations of p-adic groups. I will describe an invariant that is closely related to the "minimal K-type" of a cuspidal representation. As an application, one can show that there is a direct relationship between the depth of a representation and the slope of a singular point. This talk is based on joint work with D. Sage.

Daniel Brice (Auburn) **Derivations of parabolic subalgebras of reductive Lie algebras**

Let $\mathcal{F}$ be a field, characteristic zero and algebraically closed or $\mathbb{R}$. Let $\mathfrak{g}$ be a reductive Lie algebra over $\mathcal{F}$. Let $\mathfrak{q}$ be a parabolic subalgebra of $\mathfrak{g}$. We characterize $\operatorname{Der}\mathfrak{q}$ as a direct sum of two ideals, and we summarize related work in the literature.

Merrick Brown (UNC Chapel Hill) **The saturated tensor product semigroup for rank 2 affine Kac-Moody algebras**

Let $L(\lambda)$, $L(\mu)$, and $L(\nu)$ be integrable highest-weight representations of $\mathfrak{g}$, a rank 2 affine Kac-Moody algebra, so that $\lambda+\mu+\nu$ is an element of the root lattice. We give simple conditions when $L(N\nu) \subset L(N\lambda) \otimes L(N\mu)$ for $N > 0$ in terms of the cohomology of G/P when P is a maximal parabolic subgroup of the Kac-Moody group associated to $\mathfrak{g}$.

We approach the tensor product decomposition problem by computing the characters in terms of string functions and using the Weyl–Kac character formula to arrive at branching functions for $\mathfrak{g} \hookrightarrow \mathfrak{g} \oplus \mathfrak{g}$. We then utilize the action of the Virasoro algebra on $L(\lambda) \otimes L(\mu)$ given by the Sugawara construction, as discussed in [Kac–Wakimoto, Adv. in Math. 70], to interpret these branching functions as characters of unitarizable Virasoro modules. This allows us to explicitly produce inequalities for the saturated tensor cone as well as saturation factors for $A_1^{(1)}$ and $A_2^{(2)}$.

Andrew Conner (Wake Forest) **Twisted matrix factorizations**

A matrix factorization of an element x in a commutative ring A is an ordered pair of maps of free A-modules $(\phi : F \to G, \psi : G \to F)$ such that $\psi\phi = xI_F$ and $\phi\psi = xI_G$. Eisenbud established a correspondence between equivalence classes of reduced matrix factorizations of x over A and isomorphism classes of nontrivial periodic minimal free resolutions (of maximal Cohen-Macaulay modules) over $A/(x)$.

In this talk, we describe progress toward a theory of noncommutative matrix factorizations. Specifically, we define a twisted matrix factorization of an element x in a noncommutative ring A, prove twisted matrix factorizations give rise to periodic free resolutions, and give examples of rings over which twisted matrix factorizations exist in relative abundance. We also discuss a version of Eisenbud's correspondence for twisted matrix factorizations.

Sam Gunningham (Northwestern) **Derived Springer Theory in Families**

The Springer correspondence can be phrased as an equivalence between a certain subcategory of perverse sheaves on the unipotent cone of a semisimple algebraic group G, and representations of the Weyl group, W. I will explain how this can be generalized to an equivalence between a certain subcategory of the G-equivariant derived category of $\mathcal{D}$-modules on G and the $(W \# H)$-equivariant derived category of $\mathcal{D}$-modules on the maximal torus

H. The subcategory and equivalence described also admit a description in terms of Hamiltonian reduction and a Harish-Chandra homomorphism. Restricting to the subcategory of $\mathcal{D}$-modules supported on the unipotent cone, we obtain a version of a recent theorem due to Rider.

Ben Harris (LSU) **Wave front sets of reductive Lie group representations**

First defined by Hörmander, the wave front set of a distribution gives a local description of its singularities. The wave front set is a fundamental notion in microlocal analysis; its algebraic cousin, the singular support (also sometimes called the characteristic variety), is a fundamental notion in the theory of algebraic $\mathcal{D}$-modules.

The wave front set of a unitary Lie group representation was first defined in a special case by Kashiwara-Vergne and in general by Roger Howe. It associates to every unitary Lie group representation a closed, invariant cone in the dual of the Lie algebra.

In this talk, we will give a geometric description of the wave front set of any unitary representation of a reductive Lie group that is weakly contained in the regular representation. For the group $SL(2, \mathbb{R})$, we will illustrate this result with pictures. If time permits, we will give applications to branching problems. This is joint work with Hongyu He and Gestur Ólafsson.

Garrett Johnson (Catholic University of America) **FRT-bialgebras and quantum Schubert cells**

The aim of this talk is to draw connections between quantum Schubert cell algebras and FRT-bialgebras. The universal bialgebra construction of Faddeev, Reshetikhin, and Takhtajian is an approach to obtaining the quantized coordinate ring of an algebraic group. I will describe how special types of quantum Schubert cell algebras, namely those which are quantizations of nilradicals of cominuscule parabolics of simple finite dimensional Lie algebras, and quotients thereof, map isomorphically onto distinguished subalgebras of FRT-bialgebras. This is joint work with Christopher Nowlin.

Liping Li (UC Riverside) **A generalized Koszul theory**

Let A be a graded locally finite algebra where A_0 is a finite-dimensional algebra whose finitistic dimension is 0. We develop a generalized Koszul theory preserving many classical results as the Koszul duality, and show an explicit correspondence between this generalized theory and the classical theory. Applications in representations of certain categories, modular skew group algebras, and extension algebras of standard modules of standardly stratified algebras will be described.

Greg Muller (LSU) **F-regularity and cluster algebras**

This talk will consider reductions of cluster algebras to fields of odd characteristic. Algebras in positive characteristic have the Frobenius endomorphism, which sends every element to its p-th power. We show that the Frobenius map on an upper cluster algebra admits a standard splitting. When the cluster algebra is locally acyclic, this Frobenius splitting is regular; that is, it does not x any non-trivial ideals. This has geometric consequences for locally acyclic cluster algebras over $\mathbb{Z}$; in particular, they have (at worst) rational singularities.

Laura Rider (LSU) **Parity sheaves on the affine Grassmannian and the Mirković–Vilonen conjecture**

We prove the Mirković–Vilonen conjecture: the integral local intersection cohomology groups of spherical Schubert varieties on the affine Grassmannian have no p-torsion, as long as p is outside a certain small and explicitly given set of prime numbers.

Dylan Rupel (Northeastern) **Generalized Feigin Homomorphisms**

In the early '90s Feigin proposed the existence of a family of homomorphisms from a quantized enveloping algebra to rings of skew-polynomials, these "Feigin homomorphisms" have proven useful for studying the fraction field of the quantized enveloping algebra. Also in the early '90s Ringel discovered an embedding of the quantized enveloping algebra into the Hall-Ringel algebra of a hereditary category. In an ongoing work with Arkady Berenstein we extended the Feigin homomorphisms to the entire Hall-Ringel algebra and investigated applications to quantum cluster algebra structures on quantum unipotent groups.

Another attempt to gain a concrete hold of the quantized enveloping algebra embeds it into a quantum shuffle algebra. My goal in this talk will be to complete the resulting tetrahedron of maps. I will extend the Feigin homomorphisms to the quantum shuffle algebra and define a "quantum shuffle character" from the Hall-Ringel algebra to the quantum shuffle algebra making the diagram commute. Time permitting I will discuss applications to quantum cluster algebras.

Salvatore Stella (Northeastern) **Wonder of sine-Gordon Y-systems**

The sine-Gordon Y-systems and the reduced sine-Gordon Y-systems were introduced by Tateo in the 90s in the study of the integrable deformation of conformal field theory by the thermodynamic Bethe ansatz method. The periodicity property and the dilogarithm identities concerning these Y-systems were conjectured by Tateo, and recently proved using cluster algebras. In this talk we explain how these Y-systems can be understood using triangulations of polygons and how this provide automatically a proof of both periodicity and dilogarithm identities in full generality. This is a joint work with T. Nakanishi.

Andrew Talian (Georgia) **Endotrivial modules for Lie superalgebras**

A supermodule M for a Lie superalgebra is endotrivial if $\mathrm{End}_k(M) = k \oplus P$ for some projective supermodule P. Such modules form a group under the tensor product. In this talk, we classify the group of endotrivial modules for certain Lie superalgebras of particular interest and show that in some cases there are finitely many endotrivials of a fixed dimension n.

Mary Clair Thompson (Auburn) **Asymptotic Behavior of the Aluthge Iteration**

In 2011, Antezana et. al. showed that the iterated Aluthge sequence converges on all of $\mathbb{C}_{n\times n}$. We consider a related question on noncompact, semisimple Lie groups and show that group level Aluthge sequence converges in this context.

María Vega (NC State) **Twisted exponents and twisted Frobenious–Schur indicators for Hopf algebras**

The classical FrobeniusSchur (FS) indicators for finite groups are character sums defined for any representation and any integer $m \geq 2$. In the familiar case $m = 2$, the FS indicator partitions the irreducible representations over the complex numbers into real, complex, and quaternionic representations. In previous work, we constructed twisted versions of FS indicators for semisimple Hopf algebras which were introduced by Linchenko and Montgomery. In this talk, I will introduce a twisted version of the exponent of a module generalizing the results of Etingof and Gelaki. I will also explain its relationship to twisted FS indicators. This is joint work with Daniel Sage.