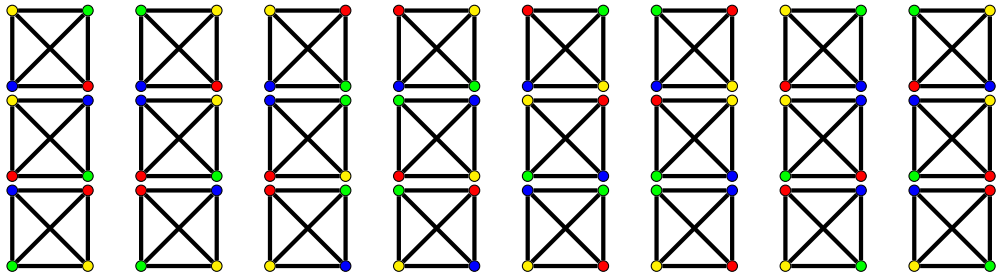
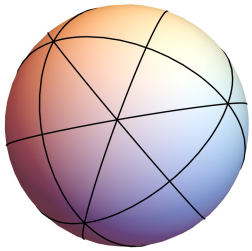
The polynomial $P(k) = -6k + 11k^2 - 6k^3 + k^4$ tells you ...

... the number of ways the complete graph on 4 vertices can be k-colored. For instance, there are $P(4) = 24$ different 4-colorings.



... the number of disjoint regions on the unit sphere where all three coordinates are different. There are $P(-1) = 24$ of them (18 visible below).



The sequence $(a_1, a_2, a_3, a_4) = (6, 11, 6, 1)$ of absolute values of the coefficients of $P(k)$...

... is unimodal: there is an index i for which

$$a_1 \leq \cdots \leq a_{i-1} \leq a_i \geq a_{i+1} \geq \cdots \geq a_4.$$

For this polynomial, $i = 2$ and $6 \leq 11 \geq 6 \geq 1$.

... is log-concave: for each j with $1 < j < 4$, we have $a_{j-1}a_{j+1} \leq a_j^2$. For this polynomial, this says $6 \cdot 6 \leq 11^2$ and $11 \cdot 1 \leq 6^2$.

If you find any of these phenomena interesting, join us in Fall 2022 for

MATH 4997-2

Geometry and Combinatorics of Polynomials

where we will explore polynomials like this one, and try to understand why they have properties such as unimodality and log-concavity.

For more information, contact Christin Bibby or Dan Cohen.