MATH 4039-1 **Midterm Review** **Spring 2022**

The midterm exam will take place on Thursday March 24, covering material we will have discussed from Chapters 1 and 2 in the text by then (probably through §2.6, including some matters (re)visited in §2.7).

A (likely non-comprehensive) list of concepts etc. you should know and be ready to work with is below.

If there are particular aspects of this material you would like to (further) discuss, make me aware of them prior to Monday March 21 – I may be able to include them in the discussion in class on Tuesday March 22. You may also make use of my office hours, and/or ask questions by email.

I anticipate that (some of the) problems on the exam will be of a similar nature to those you've encountered in the homework (both collected and uncollected), examples and proofs we've discussed in class, etc.

Some concepts, results, definitions... you should know, be able to prove...

Know the definitions of *open set*, *limit point*, *closed set* (in $\mathbb{R}^n$) (pages 1, 2)

A subset of $\mathbb{R}^n$ is closed if and only if its complement is open (Proposition 1.1.2)

Be prepared to find limit points, determine of a given set is open, closed, both, or neither (e.g., Exercise 1.1.4)

The relationship between limit points and convergence of sequences in $\mathbb{R}^n$ (Proposition 1.1.6)

An arbitrary union of opens in $\mathbb{R}^n$ is open; a finite intersection of opens in $\mathbb{R}^n$ is open (Exercises 1.1.7, 1.1.8)

For a set X, know the definition of of a *topology on* X (page 3)

If A is a subset of $\mathbb{R}^n$ (or more generally, of a topological space X), what is the subspace topology on A? A subset $U \subset A$ is open in A if ..., a subset $C \subset A$ is closed in A if ... (page 4)

If X is a topological space, and $\sim$ is an equivalence relation on X, what is the *quotient topology* on $X/\sim$?
What are open sets in $X/\sim$ (in this topology) (page 9)?

A topological space X is *Hausdorff* if ... (page 11)
$\mathbb{R}^n$ is Hausdorff, a subspace A of $\mathbb{R}^n$ is Hausdorff (Exercises 1.1.30, 1.1.31)

For topological spaces X and Y, a function $f\colon X \to Y$ is *continuous* if ... (page 13)
Know the relationship between this topological definition of continuity and the "calculus" definition (also page 13), and the relationship between continuity and sequences for subspaces of Eulcidean space (Proposition 1.2.2)
Be prepared to work with continuity, examples of continuous functions, etc. (e.g., pages 14, 15...)

What is a *homeomorphism*? What does "X and Y are *topologically equivalent*" mean? (page 19)
Be prepared to work with these notions (as in Exercises 1.2.20–1.2.24)

What is a *topological property*? List the topological properties we have encountered, with definitions and proofs that they are in fact topological properties (pages 22–31, and elsewhere)
Be able to use topological properties to distinguish topological spaces (as in various Exercises, Examples in §1.3...)

A topological space X has the *fixed point property* if ...
Exhibit examples of spaces which have the fixed point property, which do not have the fixed point property

A topological space X is *path connected* if ...
Exhibit examples of spaces which are path connected, which are not path connected

Know and be able to use the *Jordan separation theorem* (page 26, and elsewhere)

A topological space X is *compact* if ...
Exhibit examples of spaces which are compact, which are not compact
What does the *Heine-Borel theorem* say? (page 28, subsequent discussion page 29)

Know, and be prepared to work with, the definitions of a *surface* (page 36), a *surface-with-boundary* (page 37)
Illustrate the difference between these two notions with examples

We have, by now, encountered many (compact, path-connected) surfaces. Give examples, with explanations of why they are topologically distinct when possible.

Be comfortable working with the connected sum of surfaces (introduced in §2.2)

Be comfortable working with plane models of surfaces, with the connected sum construction in this context (§2.3)...
Be able to show that a given plane model represents a surface (Exercise 2.3.2, the discussion on page 48...)

A surface (or surface-with-boundary) is *non-orientable* if ..., is *orientable* if ... (§2.4)
How are these notions related to the connected sum construction? (Proposition 2.4.1)
How can non-orientability be detected in a plane model? (Proposition 2.5.1)
Exhibit examples of orientable/non-orientable surfaces, be comfortable working with simple closed curves and "cut-and-paste" methods in (the plane models of) these, and implications (various exercises in §2.3)

State the *Classification Theorem for Surfaces*, and its Corollary regarding plane models (§2.6, proof in §2.7)
If a surface "comes to you" as a connected sum or as a plane model, can you identify it in terms of this theorem? For instance, explain why $\mathbb{K} \cong 2\mathbb{P}$. See also various exercises in §2.6. In particular, the fact that the surfaces $\mathbb{P}\#\mathbb{T}$ and $3\mathbb{P}$ are homeomorphic ($\mathbb{P}\#\mathbb{T} \cong 3\mathbb{P}$ as discussed in class), which is what Exercise 2.6.4 (a) is getting at, is useful in this context.