MATH 7590–1 Arrangements of Hyperplanes Spring 1998
MWF 2:30–3:30 113 Lockett

> Show that n cuts can divide a cheese into as many as $(n+1)(n^2-n+6)/6$ pieces.
> Problem E 554. *Amer. Math. Monthly* **50** (1943), p. 59.

An arrangement of hyperplanes is a finite collection of codimension one affine subspaces of a finite dimensional vector space over some field. Arrangements arise in a number of branches of mathematics: the study of braids, reflection groups, singularities, and combinatorics to name a few. Techniques from algebra, combinatorics, algebraic geometry, and topology may be used in their study. Arrangements are easily defined, and yield many connections between the various subjects mentioned above.

We begin by introducing several combinatorial and algebraic tools used in the study of arrangements. The intersection poset $L(\mathcal{A})$ is an important combinatorial invariant of an arrangement $\mathcal{A}$. We define the Möbius function of $L(\mathcal{A})$, study its properties, and use it to define the Poincaré polynomial of $L(\mathcal{A})$. We also use $L(\mathcal{A})$ to construct a graded algebra $A(\mathcal{A})$, and discuss the structure of this algebra in terms of the combinatorial invariants at hand. A key technical tool is the method of deletion and restriction, which allows induction on the number of hyperplanes in the arrangement.

A fundamental topological invariant of an arrangement $\mathcal{A}$ is the complement $M(\mathcal{A})$. If $\mathcal{A}$ is a real arrangement, the complement is a disjoint union of convex sets called chambers. A natural question to ask (generalizing the above problem) is: Given $\mathcal{A}$, how many chambers? Using the combinatorial tools introduced above, we can give a complete answer to this question. If $\mathcal{A}$ is a complex arrangement, the complement is an open, smooth manifold. In this instance, we can study topological invariants such as the fundamental group, the homology, the cohomology algebra, and so on. Many of these topological invariants of an arrangement are computable in terms of the combinatorial invariants of the arrangement. For instance, the Poincaré polynomial of the complement coincides with that of $L(\mathcal{A})$, and the algebra $A(\mathcal{A})$ provides a combinatorial description of the cohomology of $M(\mathcal{A})$.

The relationship between the combinatorics and more refined topological invariants of arrangements is not yet well understood. We will pursue a number of these topological invariants, including the fundamental group, the braid monodromy, and further invariants obtainable from them. Depending on the interests and background of the audience, we will also discuss several more specialized topics, including braid groups, (stratified) Morse theory, Alexander invariants, the lower central series...

Much of the basic material may be found in *Introduction to Arrangements* by P. Orilk (CSBM Lecture Notes **72**, Amer. Math. Soc., 1989), which is quite accessible (and quite reasonably priced!). A more complete reference is *Arrangements of Hyperplanes* by P. Orlik and H. Terao (Grundlehren **300**, Springer-Verlag, 1992).

Little or no background is necessary for the combinatorial aspects of the discussion. The topological aspects require some familiarity with the basic notions of algebraic topology (e.g. MATH 7520). Students who have not taken 7520, but have taken MATH 7512, will be able to appreciate most aspects of the course.

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