1. (a) Solve the initial value problem $\dfrac{dy}{dx} = y^2 \sin x, \quad y(0) = 1.$

   $y = \sec x$ solves the IVP

   (b) Does the initial value problem in part (a) have a unique solution? Explain.

   It does. Use the existence and uniqueness theorem for first-order IVPs.

2. Find the general solution of each of the following differential equations.

   (a) $y' + 4y = e^{-x}$ $\quad$ $y = \dfrac{1}{3}e^{-x} + Ce^{-4x}$

   (b) $y'' + 10y' + 25y = 0$ $\quad$ $y = c_1 e^{-5x} + c_2 x e^{-5x}$

   (c) $y'' - 2y' + 5y = 0$ $\quad$ $y = c_1 e^{x} \cos 2x + c_2 e^{x} \sin 2x$

3. (a) Solve the initial value problem $y'' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 6.$

   $y = 2e^{2x} - e^{-2x}$

   (b) Find a particular solution of the differential equation $y'' - 4y = 8e^{2x} + 5\sin x.$

   $y = 2xe^{2x} - \sin x$

4. Consider the differential equation $x^2 y'' - 2xy' + 2y = 0, \quad x > 0.$

   (a) One solution of this differential equation is $y_1(x) = x$. Use the method of reduction of order to find a second linearly independent solution $y_2(x)$ on the interval $I = (0, \infty)$. Briefly explain why the solutions $y_1(x)$ and $y_2(x)$ are linearly independent on I.

   $y = Ax^2 + Bx$ is a solution by reduction of order, so $y_2 = x^2$ is a second solution

   $y_1 = x$ and $y_2 = x^2$ are not proportional on I, so are linearly independent on I

   This can also be established using the Wronskian: $W[x, x^2] = x^2 \neq 0$ for $x > 0$

   (b) Find the general solution of this differential equation.

   $y = c_1 x + c_2 x^2$

5. Newton's law of cooling states that the rate of change of the temperature of an object with respect to time t is proportional to difference between the temperature, $W(t)$, of the object at time t and the temperature, R, of the surrounding medium.

   Suppose that the temperature of the water in my cup is 45°F at the beginning of class (at time $t = 0$), and that room temperature $R = 72$°F is constant. To determine the temperature of my water at the end of class using Newton's law of cooling, you would have to solve a certain initial value problem. Write this initial value problem down.

$$\frac{dW}{dt} = k(W - 72), \quad W(0) = 45$$