1. (a) Solve the initial value problem $\dfrac{dy}{dx} = 4x\sqrt{y-1}$, $y(2) = 1$.
   Does this initial value problem have a unique solution? Explain

   (b) Find an initial condition for which the initial value problem $\dfrac{dy}{dx} = 4x\sqrt{y-1}$, $y(x_0) = y_0$
   is certain to have a unique solution.

2. Solve the following first-order differential equations.

   (a) $y' - y\tan x = 8\sin^3 x$ (c) $xy' - y = 4xy^2$ Hint: make the substitution $u = y^{-1}$

   (b) $(x + xy^2) + e^{x^2}y\dfrac{dy}{dx} = 0$ (d) $\dfrac{dy}{dx} = \dfrac{3y}{3x - y}$ Hint: make the substitution $V = \dfrac{y}{x}$

3. For the following, find the general solution of the differential equation, and then solve the initial value problem.

   (a) $y'' - 6y' + 9y = 0$, $y(0) = 3$, $y'(0) = 0$ (c) $y'' - 4y' + 7y = 0$, $y(0) = 3$, $y'(0) = 0$

   (b) $6y'' - 5y' + y = 0$, $y(0) = 3$, $y'(0) = 0$ (d) $y'' + 2y' + y = 2\cos(x)$, $y(0) = 3$, $y'(0) = 0$

4. Find the general solution of each of the following non-homogeneous differential equations.

   (a) $y'' - 2y' - 3y = 3x^2 - 5$ (b) $y'' - 2y' - 3y = 16e^{-x}$ (c) $y'' - 2y' - 3y = 10 - 6x^2 - 4e^{-x}$

   (d) Explain why you can use the results of parts (a) and (b) to do part (c).

5. Consider the differential equation $xy'' - (x + 1)y' + y = 0$, $x > 0$.

   (a) Verify that the function $y_1(x) = e^x$ is a solution of this differential equation.

   (b) Use the method of reduction of order to determine a second solution.

   (c) Find the general solution of this differential equation on the interval $I = (0, \infty)$.
   Give a thorough explanation why the solution you give is the general solution.

6. There are 42 people in this class. Suppose there is a certain rumor spreading through class, and that, as of today, 5 people have heard it. Let $R(t)$ be the number of people in this class who have heard the rumor t days from now. Assume that the rate at which the rumor spreads (that is, the rate of increase of R) is equal to the product of the number of people who have heard the rumor at time t and the number of those who have not.
   To determine how many people in this class had heard the rumor, say one week from now, you would first have to solve a certain initial value problem. Write this initial value problem down, then solve it.

7. Boyle-Mariotte's law for ideal gases. For a gas at low pressure p (and constant temperature), the rate of change of the volume $V = V(p)$ is equal to $-V/p$. Determine the volume as a function of p.