**MATH 2090** **Review Problems for Exam 2** **Fall 2000**

1. Consider the system of linear equations
$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & 3x_3 & + & x_4 & = & 6 \\ 2x_1 & - & x_2 & + & 3x_3 & - & x_4 & = & 3 \\ -x_1 & + & x_2 & - & 2x_3 & & & = & -3 \end{array}$$
Write this system in vector form, find the rank of the coefficient matrix, and find the solution set of the system.

2. Consider the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$ and the constant vector $\vec{b} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$.

   (a) Use Gauss-Jordan elimination to find the inverse of $A$.

   (b) Use your answer from part (a) to solve the system of equations $A\vec{x} = \vec{b}$.

   (c) Is the transpose of $A$ non-singular? Explain.

3. Consider the matrix $A = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 3 \\ 0 & 1 & -1 & 0 \\ 1 & 3 & -2 & 4 \end{bmatrix}$.

   (a) Compute the determinant of $A$. Does the matrix $A$ have an inverse? Explain.

   (b) If $B$ is a $4 \times 4$ matrix with $\det(B) = -7$, what is the determinant of the product $AB$? Explain.

   (c) Can a system $A\vec{x} = \vec{b}$ involving this matrix $A$, and any constant vector $\vec{b}$, be inconsistent? Explain.

4. Consider the homogeneous system of linear equations
$$\begin{array}{rcrcrcr} (1-\lambda)x_1 & + & 2x_2 & + & 6x_3 & = & 0 \\ & & (2-\lambda)x_2 & + & 3x_3 & = & 0 \\ & & x_2 & + & (4-\lambda)x_3 & = & 0 \end{array}$$

   (a) Find all values of $\lambda$ for which this system has infinitely many solutions.

   (b) Find the solution set of this homogeneous system when $\lambda = 1$.