MATH 2090 **Review Problems for Exam 4** **Fall 2000**

1. For each of the matrices A below, find a fundamental set of solutions (explain why the solutions you find form a fundamental set), a fundamental matrix, and the general solution of the system $\vec{x}\,' = A\,\vec{x}$. In each case, determine if the matrix A is defective or diagonalizable. If A is diagonalizable, find an invertible matrix S and a diagonal matrix D so that $S^{-1}AS = D$. Also, solve the initial value problem indicated in part (a).

(a) $A = \begin{bmatrix} 1 & 2 \\ 8 & 1 \end{bmatrix}$, $\vec{x}(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 4 & -5 \\ 1 & 2 \end{bmatrix}$ (c) $A = \begin{bmatrix} 7 & -4 \\ 4 & -1 \end{bmatrix}$ (d) $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$

(e) $A = \begin{bmatrix} 2 & -3 & 2 \\ -1 & 0 & 2 \\ -1 & -3 & 5 \end{bmatrix}$

2. The differential equation $x^3y''' + x^2y'' - 2xy' + 2y = 0,\ x > 0$ has three solutions of the form $y(x) = x^r$. Find these solutions, show that they are linearly independent on the interval $(0, \infty)$, and find the general solution of this differential equation.

3. For each of the non-homogeneous linear differential equations $P(D)y = F(x)$ below, find the complementary solution, find the annihilator of the function $F(x)$, and determine the form of a particular solution. In parts (a) and (b), also find the general solution of the differential equation, and solve the initial value problem in part (a).

(a) $(D-1)(D-2)(D-3)y = 6e^{4x},\ y(0) = 4,\ y'(0) = 10,\ y''(0) = 30$ (b) $y''' + 3y'' + 3y' + y = 2e^{-x} + 3e^{2x}$

(c) $(D^2 - 2D + 2)(D-3)^2(D+2)y = x^2 - e^x \cos x + 3e^{3x}$

4. For each of the following functions, find the Laplace transform. In parts (a) and (b), use the definition to determine the Laplace transform of the function.

(a) $f(t) = e^{-3t}$ (b) $g(t) = \sin 2t$ (c) $h(t) = 2e^{-3t} - 3\sin 2t$

5. Find the inverse Laplace transform of each of the following functions. (You may use the table on page 543.)

(a) $F(s) = \dfrac{3}{s-2}$ (b) $G(s) = \dfrac{1}{s^2+4}$ (c) $H(s) = \dfrac{4}{s^2} - \dfrac{s+2}{s^2+9}$ (d) $I(s) = \dfrac{1}{s(s-1)}$