Math 2090 **Final Exam Information** **Fall 2001**

Final Exam: Saturday, December 15, 12:30 – 2:30 p.m., in 232 Coates

Office Hours: Wednesday, December 12, 11:00 a.m. – 1:00 p.m., in 372 Lockett
Thursday December 13, 1:30 – 3:30 p.m., in 372 Lockett

Review Session: Friday, December 14, 3:00 – 5:00 p.m., in 112 Lockett

You may also contact me by email at `cohen@math.lsu.edu` at any time.

Materials:
All homework assignments, review sheets, and answers to the problems on the exams are posted. Follow the links from `http://www.math.lsu.edu/~cohen/courses/FALL01/M2090syl.html`. My answers to the exam problems and the problems on the review sheets are posted on the exams page, as are the review sheets themselves. The only "new" topic you will encounter on the final is the Laplace transform, and the solution of initial value problems using it. A couple of representative problems, with answers, are included at the end of this sheet. Other suitable review problems may be found in the examples and homework assignments from Chapter 9 (sections 1, 2, 4, and 5). The final will be cumulative, and will address topics we've covered this semester in a proportionate and appropriate manner. A list of relevant topics, and some comments concerning them, is included below.

Chapter 1. First-Order Differential Equations
Be comfortable with the terminology [§1.2]. The main classes we discussed are separable [§1.4 – separate variables, then integrate] and linear [§1.6 – integrating factor] first order differential equations. I won't ask about existence/uniqueness theorems, other classes of 1st order ODEs, or applications on the final.

Chapter 2. Second-Order Linear Differential Equations and
Chapter 7. Linear Differential Equations of Order n
Be able to solve constant coefficient linear ODE's, by finding the roots of the auxiliary polynomial for homogeneous (or complementary) solutions [§2.3, §7.2], and by using annihilators/the method of undetermined coefficients to find particular solutions of non-homogeneous equations [§2.4, §7.3]. You should be able to produce real-valued solutions (using Euler's formula if need be, see §2.3). Be aware of the underlying theory [§2.1, §7.1]. I will not ask you about existence/uniqueness theorems (in this context, or in the context of a 1st order DE), but you should, for instance, know what a solution of a DE is, understand that a homogeneous linear nth order ODE has n linearly independent solutions, be able to determine if solutions are independent (see §5.5), ...

Chapter 3. Matrices and Systems of Linear Algebraic Equations
I will not explicitly ask you about things like matrix algebra [§3.1, §3.2] or terminology for systems of linear equations [§3.3] on the final. I will assume you are proficient at matrix operations: can multiply matricies, take transposes, know what the identity matrix is, can translate a system of equations into matrix form, etc.... Topics I may ask you about include: row operations, echelon form, rank, Gaussian elimination, finding the solution set of a system of linear equations, homogenous systems, inverse of a square matrix [§3.4 – §3.6]. Be able to carry out the algorithms we developed to deal with these. Also be able to interpret your results, and understand their implications (see e.g., Ch. 5 below). For instance, if you know the rank of a matrix A, what can you say about systems involving A? If A is square, what are the relations between $\operatorname{rank} A$, systems involving A, the invertibility of A, $\det A$, etc.?

Chapter 4. Determinants
You need to be able to compute determinants using various properties, or cofactor expansion, or some combination. These are summarized in §4.4, and are discussed in more detail in §4.2 and §4.3. Understand the implications of your calculations. For instance, what is the relation between $\det A$ and the invertibility of A? between $\det A$ and systems of linear equations with coefficient matrix A? Note that I can implicitly ask you to compute determinants: "Find the eigenvalues..."

Chapter 5. Vector Spaces
I won't ask you to work with the definition of a vector space [§5.2], but you should be aware of what one is (informally, a set of "vectors" which we can add, and multiply by scalars). I may ask you if a subset of a "known" vector space is a subspace [§5.3]. "Known" vector spaces you may encounter include $\mathbb{R}^n$, P_n – polynomials of degree $< n$, $C^k(I)$ – functions with k derivatives on I, $M_n(\mathbb{R})$ – $n \times n$ matrices with real entries, ... You should be able to work with various vector space notions: linear combinations, span [§5.4]; linear dependence and independence [§5.5] (and independence of functions and the Wronskian, discussed in the context of linear ODEs in Ch. 2 and Ch. 7 and systems in Ch. 8); basis and dimension [§5.6]. Questions regarding these notions can often be reduced to questions about systems of linear equations [Ch. 3], so be prepared to interpret the latter in this way.

Chapter 6. Linear Transformations and the Eigenvalue/Eigenvector Problem
Know what a linear transformation is, and be able to determine if a given mapping between vector spaces is one [§6.1]. You should also know what the kernel and range of a linear transformation are, and be able to find bases for them [§6.3]. This problem – find the kernel of a linear transformation – sums up much of the course. Think about why. Be able to find the eigenvalues, eigenvectors, eigenspaces of a square matrix A, to determine if A is defective [§6.5, §6.6], and to determine if A is diagonalizable [§6.7]. As with determinants, I can ask you to solve the eigenvalue problem implicitly: "Find the general solution of the system $\vec{x}\,' = A\,\vec{x}$."

Chapter 8. Systems of Differential Equations
Be able to solve homogeneous constant coefficient systems, with diagonalizable coefficient matrix [§8.5] and with defective coefficient matrix [§8.6]. For the latter, problem 6 (a) on Exam 4, problem 1 (c) on the reveiw for this exam, and §8.6, #1 – 4, 11, 14, 15, give a reasonable indication of the sort of problem you might encounter in this context. As in Ch. 2 and Ch. 7, if you're given a real system $\vec{x}\,' = A\,\vec{x}$, you should be able to generate real-valued solutions (see §8.5 and for instance problem 6 (b) on Exam 4). Also, be comfortable with the relevant terminology (e.g. solution, fundamental solution set, fundamental matrix, etc.), and be aware of the underlying theory [§8.2 – §8.4]. For instance, under what conditions is a set of solutions a fundamental set of solutions? What is the Wronskian in this context? What is it's use?

Chapter 9. The Laplace Transform ...
Be able to solve constant coefficient linear initial value problems using the Laplace transform [§9.4, §9.5]. For the latter, among other things, you will need to be able to carry out partial fractions decompositions (reviewed in Appendix 2). While one of the strengths of the Laplace transform is its ability to handle IVPs with discontinuous forcing function [§9.7], I will not ask you about this on the final. I will provide a table of Laplace transforms and the first shifting theorem. I will not provide the relation between the Laplace transform of a function and its derivative – this you should commit to memory. You should also know the definition and properties of the Laplace transform [§9.1], be able to calculate Laplace transforms using the definition, be able to calculate Laplace transforms and inverse Laplace transforms of functions using a table, etc..

Use the definition of the Laplace transform to find the Laplace transform of the function $f(t) = e^{-3t}$.

$$L[e^{-3t}] = \int_0^\infty e^{-st}e^{-3t}\,dt = \lim_{N\to\infty}\int_0^N e^{-(s+3)t}\,dt = \lim_{N\to\infty} \left.\frac{-e^{-(s+3)t}}{s+3}\right|_0^N = \lim_{N\to\infty}\left[\frac{1}{s+3} - \frac{e^{-(s+3)N}}{s+3}\right] = \frac{1}{s+3}$$

provided $s > -3$

Use the Laplace transform to solve the initial value problem $y'' + y' - 2y = 3e^{-2t}$, $y(0) = 3$, $y'(0) = -1$.

$$L[y''] + L[y'] - 2L[y] = 3L[e^{-2t}] \qquad s^2L[y] - sy(0) - y'(0) + sL[y] - y(0) - 2L[y] = \frac{3}{s+2} \qquad \text{let } Y = L[y]$$

$$(s^2+s-2)Y - 3s - 2 = \frac{3}{s+2} \qquad Y = \frac{3s+2}{s^2+s-2} + \frac{3}{(s+2)(s^2+s-2)} = \frac{2}{s-1} + \frac{1}{s+2} - \frac{1}{(s+2)^2}$$

So $y(t) = L^{-1}\left[\frac{2}{s-1} + \frac{1}{s+2} - \frac{1}{(s+2)^2}\right] = 2e^t + e^{-2t} - te^{-2t}$